A Systematic Formulation of the Continuous Adjoint Method Applied to Viscous Aerodynamic Design

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ABSTRACT

A continuous adjoint approach to aerodynamic design for viscous compressible flow on unstructured grids is developed, and three important problems raised in the continuous adjoint literature are solved: using tools of shape deformation of boundary integrals a generic adjoint formulation is developed with independence of the kind of mesh used; a systematic way of reducing the 2nd order derivative terms which arise is presented which avoids the need of using higher order numerical solvers to obtain accurate approximations of the 2nd order derivatives; and finally, the class of admissible optimization functionals is clarified. The accuracy of the sensitivity derivatives is assessed by comparison with finite-difference computations, and the validity of the overall methodology is illustrated with several design examples in demanding subsonic conditions.

Aerodynamic design optimization by adjoint methods (control theory) has received much attention recently since the pioneering work of Prof. Jameson[1][2]. In these methods the goal is to evaluate the response (sensitivity derivatives or gradients) of a given functional of the flow to a change of defining parameters (control or design variables): surface deformations, change in angle of attack or Mach number, etc. These gradients are then used as input for an appropriate minimization module[3].

The fundamentals of the procedure are as follows. We restrict ourselves to shape-deformation problems. The starting point is a generic functional of the form:

$$J = \int_S f\left(U, \vec{n}\right) ds$$

where S is the control surface (airfoil or aircraft surface), U are the flow variables and $\vec{n}$ is normal to S. Considering a deformation $\delta\vec{x}$ of the control surface, the following generic variation results:

$$\delta J = \underbrace{\int_{\delta S} f\left(U, \vec{n}\right) ds}_{\text{Geometric variation}} + \underbrace{\int_S \frac{\partial f}{\partial U} \delta U ds}_{\text{Flow variation}}.$$

subject to the (steady) flow equations (2D or 3D Euler or Navier-Stokes equations). Also, the geometric variation contains three terms[4][5]

$$\int_{\delta S} f\left(U,\vec{n}\right)ds = \underbrace{\int_S \frac{\partial f}{\partial U}\left(\delta\vec{x}\cdot\vec{\nabla}U\right)ds}_{\text{Displacement Term}} + \underbrace{\int_S\left(\frac{\partial f}{\partial\vec{n}}\delta\vec{n}\right)ds}_{\text{Normal Variation}} + \underbrace{\int_S f\,\delta ds}_{\text{Curvature Term}}$$

where the curvature term has the form

$$\int_S f\,\delta ds = \int_S f\,\mathrm{K}ds, \qquad \mathrm{K} = \begin{cases}\left(\partial_{tg}\delta x_{tg} - \kappa\delta x_n\right) & \text{(2D)}, \quad \kappa \quad \text{curvature of profile} \\ \left(\vec{\nabla}_{tg}\cdot\delta\vec{x}_{tg} - 2\mathrm{H}_m\delta\vec{x}_n\right) & \text{(3D)}, \quad \mathrm{H}_m \quad \text{mean curvature of surface}\end{cases}$$

As for the flow variation term, it results in terms containing flow variations not directly fixed by linearized boundary conditions such as δP or $\delta\sigma_{ij}$. Instead of solving the linearized flow equations, the standard solution is to introduce the adjoint state Ψ subject to the adjoint equations:

$$\left(\vec{A}+\vec{A}^v\right)\cdot\vec{\nabla}\Psi^T + \frac{\partial}{\partial x^i}\left(\left[D_{ji}\frac{\partial\Psi^T}{\partial x^j}\right]\right) = 0$$

$$\vec{A} = \left(\partial\vec{F}/\partial U\right)^T, \quad \vec{A}^v = -\left(\partial\vec{F}^v/\partial U\right)^T, \quad D_{ij} = \left(\partial F^v{}_i/\partial\left(\partial U/\partial x^j\right)\right)^T$$

where $\vec{F}$ and $\vec{F}^v$ are the inviscid and viscous flux vectors respectively

with the appropriate adjoint boundary conditions on *S* so as to allow the computation of the flow variation term[1][2][4][5]. The resulting expressions are suitable for optimization under viscous as well as inviscid flow conditions on both unstructured and structured grids. The procedure also allows the determination of the general structure of the functionals. For functionals defined on a control surface *S* with non-slip and adiabatic boundary conditions the structure is the following

$$\int_S g\left(\vec{f},T\right)ds, \quad \vec{f} = P\vec{n} - \vec{n}\cdot\sigma$$

Therefore, optimization is possible in this case with respect to any of the components of the total force $\vec{f}$ exerted by the fluid on the wall (including both the pressure and the viscous stress terms), as well as with respect to surface temperature distributions *T*. In particular, functions that depend solely on the pressure are allowed, a possibility that has been largely ignored in the literature.

In the variation of the above functional, and in general in the computation of sensitivity derivatives for viscous flows, second order derivatives of the flow variables on *S* are required. This is cumbersome (and has in fact been pointed out as a drawback of the approach[4]) as the accurate numerical evaluation of such derivatives requires at least third-order accuracy, which is beyond the capabilities of most unstructured flow solvers. This issue has been solved with the development of a systematic way of reducing the order of the higher derivative terms[5] which amounts to the utilization of the flow equations restricted to *S* to convert the normal derivatives on the second order terms into tangent derivatives and integrate those by parts to reduce the order of the derivatives. With this strategy, compact expressions result. For, e.g., drag/lift minimization the following expression is obtained[5]:

$$\delta J = \int_S \delta x_n G ds$$

$$G = (\vec{n}\cdot\partial_n\vec{u})\left(\rho\psi_1 + \rho H\psi_5\right) + \vec{n}\cdot\Sigma\cdot\partial_n\vec{u} - \psi_5\left(\vec{n}\cdot\sigma\cdot\partial_n\vec{u}\right) + \psi_5\left(\sigma_{ij}\partial_i u_j\right) - k\left(\vec{\nabla}_{tg}\psi_5\right)\cdot\left(\vec{\nabla}_{tg}T\right)$$

Therefore, sensitivities do not depend on the tangent part of deformation of the control surface, and the Local sensitivity G, which by definition is the direction of optimal deformation, can be used as deformation function (instead of conventional Hicks-Henne functions, for example).

As an example of the overall developments, Figs. 1 and 2 show a viscous design example carried out with the methodology developed in this work. The proposed design problem starts with a NACA0012 airfoil and flow conditions of Mach number equal to 0.3, angle of attack of 2.50º and low Reynolds number of 1000 to keep the flow laminar along the airfoil. The objective is drag minimization, increasing the lift to 0.15, using 3 geometrical constraints: minimum value for the greatest thickness (12%), frozen curvature at the leading edge and minimum thickness at 75% of the chord.

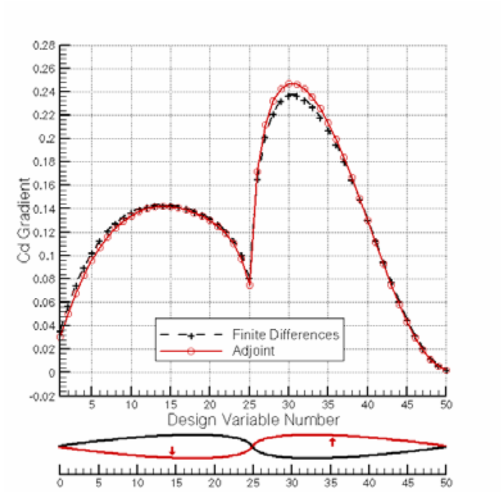

Fig. 1. Viscous C_d subsonic gradients

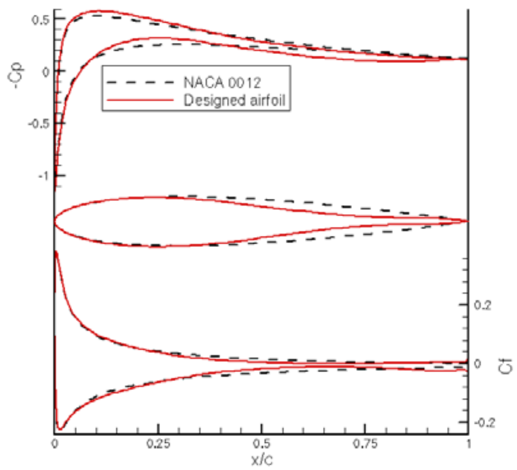

Fig. 2. Initial and designed C_p and C_f

In Fig. 1 a comparison between the gradients computed by finite-difference and adjoint methods is shown. The agreement is excellent. The results of the subsonic optimization are shown in Fig. 2. After 9 design cycles the new airfoil based on a NACA0012 has a drag of 0.1225 that is a 97% of the original NACA 0012 drag (reduction of 36 counts), while the final lift is a 111% greater than the original one.

To sum up, in this paper a breath of fresh air is supplied to the classical aerodynamic design via the continuous adjoint method. Some crucial problems are solved and new points of view and investigation lines are proposed in this work.

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