Research achievements : Enrique Zuazua

Overall, my field of expertise covers various aspects of Applied Mathematics including Partial Differential Equations (PDE), Optimal Design, Systems Control and Numerical Analysis. These interconnected fields have as goal the modelling, analysis, computer simulation and control and design of natural phenomena and engineering processes arising in several contexts of R+D+i. My work in these various disciplines has received significant international recognition, as can be seen, for instance, from the wide range of International Conferences in which I have been invited to deliver plenary talks, as well as the Journals in which I serve as member of its Editorial Boards, Scientific Committees, etc. I was also awarded the recognition of Highly Cited Researcher by ISI-Thomson in 2004 and the Spanish National Research Award, 2007. All these facts are summarized in Section 8 at the end of this document. [1]

During my career I have demonstrated the ability to expand and initiate new research fields, and to establish novel interdisciplinary approaches and a broad network of collaborators. This has influenced significantly the expansion, vigour and impact of our research in the field of PDE Control, Numerics and its interactions.

Some of my main contributions are summarized below:

• **Stabilization of systems of vibrations:** Stabilization by means of feedback mechanisms is one of the main topics of Control Theory, given its great mathematical complexity and its important applications in noise attenuation, flexible structures, etc. In my Doctoral Thesis, I introduced a new Lyapunov function approach that, by means of multiplier techniques, enables to prove the exponential decay of the solutions to wave equations dissipated on the boundary of the propagation medium. This method caused great interest in the community for its general nature, which was subsequently extended by myself and other groups to different cases, including non-linear wave equations, elasticity systems, Maxwell and Schrödinger equations, etc. As an example of the impact of my work in this field, the following papers with 172 and 160 citations in the ISI WoK, respectively, are worth mentioning [2], [3]. This analysis has been extended to the context of thermoelasticity in a series of works where a very rich structure of possible interactions between the thermal and the elastic components has been

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[1] Recently, in October 2011, the IEEE CONTROL SYSTEMS MAGAZINE published a biographic interview in the section on "People in Control" [October 2011 issue, pp. 48-50].

[2] V. Komornik and E. Zuazua (E.Z.), J. Math. Pures et appl. 69(1) (1990), 33-55.

[3] E. Z., Comm. in PDE, 15 (2) (1990), 205-235.

discovered [4]. We have also developed a quite complete theory for hyperbolic-parabolic problems reminiscent of fluid-structure interaction: [5]. In [6] we have also described the first study on the asymptotic behaviour of fluid-structure interaction systems in the simplified 1-d case.

Recently we have made a major contribution to the topic of large time asymptotics for hyperbolic systems with partial dissipation, establishing a link with the finite-dimensional control theory. This allowed us in [7] to find a new class of systems in which the decomposition of solutions in decaying components as time tends to infinity varies from the classical one in which the so-called Shizuta-Kawashima condition is fulfilled and solutions may be decomposed into an exponentially decaying component and another one decaying as the heat kernel. Our analysis shows that there are cases in which there is a third term decaying even more slowly, but this only occurs for multi-dimensional problems.

• **Convection-diffusion:** My work concerning the dynamical aspects of PDE brings together several articles on convection-diffusion equations, in which the combination of these two terms may produce unexpected non-linear effects. This is a classical subject in Physics, Engineering and Mathematics because of its many applications, and particularly in Fluid Mechanics. In [8] we developed a new methodology based on Lyapunov functionals and self-similar variables, enabling the first results on the asymptotic behaviour in various space dimensions to be obtained. More recently, in collaboration with J. Vancostenoble, we have adapted these techniques to equations with singular potentials that arise naturally in the linearization of combustion models and in which, from a mathematical perspective, Hardy inequalities play a fundamental role. We have also obtained new results on the possibility of controlling models incorporating these singular potentials both in the context of heat and wave processes. In collaboration with my former PhD student C. Cazacu we also obtained some novel results on Hardy inequalities for multi-polar singular potentials. In collaboration with D. Krejcirik we have developed a systematic analysis of the asymptotic behaviour of heat kernel in twisted cylinders.

• **Non-linear Control Systems:** One of the great challenges in PDE Control Theory is the move from linear to non-linear systems. This is also a highly relevant problem in applications, since the majority of the most realistic models for natural and technological processes and phenomena are non-linear (elasticity, fluids, structures, materials, etc.). My work has addressed these

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[4] G. Lebeau and E. Z., Archives Rat. Mech. Anal., 148 (1999), 179-231, (60 cit. ISI WoK).
[5] J. Rauch, X. Zhang, E. Z. J. Math. Pures et Appl., (9) 84 (2005), no. 4, 407-470. (24 citations ISI WoK).
[6] J. L. Vázquez and E. Z. M3AS, 16 (5) (2006), 637-678. (9 citations ISI WoK).
[7] K. Beauchard & E. Z., Arch. Rational Mech. Anal., 199 (2011) 177–227.
[8] M. Escobedo and E. Z., J. Funct. Analysis, 100 (1991), 119-161, (120 citations ISI WoK).

issues in the context of PDE. The paper [9] introduces a fixed-point method that is already regarded as classical. By using fine estimates of the cost of controlling linearized problems, it enables control results for wave equations with non-linearities with an optimal growth rate at infinity. In [10] this method is adapted to the case of strongly irreversible systems in time, such as heat equations, in which the final control condition must be relaxed in order to be converted into an approximate control condition. In paper [11] this work is completed with fine estimations on the cost of control for non-linear models, allowing the possibility of controlling some blowing up mechanisms, in particular. More recently we have extended this analysis to viscous Hamilton-Jacobi equations [12]

• **Controllability of linear PDE:** Despite of the advances on the controllability of nonlinear PDE there are still many fundamental questions that are still poorly understood even at the linear level. In the last few years we have got some fundamental results that we summarize below.

For wave-like equations, in the continuous setting, the observability constants are well known to be closely related to the so-called Geometric Control Condition requiring that all rays of Geometric Optics enter the set of observation on the given time horizon. This problem is not so well understood for heat-like equations. In that context, because of the infinite speed of propagation, observability holds in an arbitrarily small time and without any geometric condition. But because of the time-irreversibility it is well known that an exponential boundary layer arises at the initial time. In [13], using an unexpected and original transformation linking the heat and the wave equation, we prove the first sharp results in that context.

Due to the strong time-irreversibility of the heat equation the mapping associating the data of the adjoint solution leading to the control to the data to be controlled, does not necessarily preserve the regularity, as it happens for wave like equations, as proved in [14]. In [15], in the case of the heat equation, we prove that the behaviour is the opposite one and, actually, that the data for the adjoint equation are extremely irregular, with exponentially growing Fourier coefficients, even for very smooth data to be controlled.

Despite the important number of papers dealing with the control of heat like

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[9] E. Z., Ann. IHP. Analyse non linéaire. 10 (1993), 109-129 (77 cit. ISI WoK).
[10] C. Fabre, J. P. Puel and E. Z., Proc. Roy. Soc. Edinburgh, 125A (1995), 31-61, (150 cit. ISI WoK).
[11] E. Fernández-Cara and E. Z., Annales de l'IHP, Analyse non linéaire, 17 (5) (2000), 583-616 (81 citations ISI WoK).
[12] A. Porretta and E. Z., Annales IHP, Analyse non linéaire, 29 (2012), pp. 301-333.
[13] S. Ervedoza and E. Z, ARMA, 202 (2011) 975–1017.
[14] S. Ervedoza and E. Zuazua. Discrete Contin. Dyn. Syst. Ser. B, 14(4):1375--1401, 2010.
[15] S. Micu and E. Zuazua, Systems and Control Letters, 60 (2011) 406-413.

equations, very little is known in the context of hypo-elliptic models. In our paper [16] we make a first contribution to this field, analyzing the controllability of the famous Kolmogorov equation. Our results have attracted much attention and stimulated research of different groups on the control of hypo-elliptic equations. The issue of the anisotropic large time asymptotics for these models has been also investigated in a recent paper in collaboration with L. Ignat.

In [17] we build examples showing the optimality of the Carleman inequalities often used in the control of heat and wave equations, in what concerns the dependence with respect to lower order potentials.

More recently we have investigated more complex control patterns, often relevant in applications. In [18] we develop a new theory showing that classical duality tools can also be used to build time-switching controls fulfilling a suitable minimality condition. In a more recent paper in collaboration with Q. Lu we show that, in some instances, the control strategy can be build so to be robust with respect to unknown switching control patterns, a fact that is extremely relevant in applications.

• **Waves in heterogeneous media:** Another subject on which I have worked intensively since 1989, motivated by control theoretical questions, is the propagation of waves in highly heterogeneous media, a field of great importance, particularly in the areas of oil prospecting, acoustics, new materials, etc. In paper [19] we developed a fine construction of a fractal nature that enables one to prove that the usual properties of wave propagation and controllability are lost in the framework of Hölder continuous coefficients, thereby demonstrating the optimality of the results for coefficients of bounded variation, that we proved previously. This also establishes the limits of the existing dispersive estimates for clusters of eigenfunctions by Smith and Sogge, which play a critical role when solving nonlinear waves and Schrödinger equations in manifolds and bounded domains. I have also intensively studied the problem of wave propagation on networks, of primary importance in so many applications as irrigation, traffic, etc. Our contribution in the field is summarized in my Springer 2006 book in collaboration with R. Dáger and my recent survey article to be published in 2012 in a CIME-Springer volume.

• **Numerical Analysis:** My team is very interested in the interaction between numerical simulation, wave propagation and control, which is of great use in

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[16] K. Beauchard and E. Z., Ann. Institut Henri Poincaré, Analyse Non Linéaire, 26 (5) (2009), 1793-1815.
[17] Th. Duyckaerts, X Zhang, E. Z., Annales IHP, 25 (2008) 1–41. (18 citations ISI WoK).
[18] E. Z., J. Eur. Math. Soc., 2011, 13, 85–117.
[19] C. Castro and E. Z., Archive Rational Mechanics and Analysis, 164 (1) (2002), 39-72, 22 cit. ISI WoK

problems concerning noise attenuation, flexible structures, etc. In these problems numerical simulations rely on discrete models which, in particular, may generate spurious solutions that dramatically modify the propagation and, consequently, control properties. In 2005, a review paper published in the prestigious SIAM Review [20] summarized our work since 1998. First, we rigorously demonstrate that classical methods may give rise to erroneous numerical approximations in control because of the presence of high spurious frequencies. Secondly, we develop different methods to avoid these spurious numerical solutions by means of dual mesh filtering techniques, mixed finite element methods, numerical viscosity, etc. A series of highly complex analytical problems are also posed. They have motivated much work that is currently being performed by different research teams. Later in [21] we have shown that the same analysis applies to other relevant issues, as for instance the efficiency of the method of Perfectly Matched Layers (PML), so relevant in wave computing and simulation. My invited section lecture at International Congress of Mathematicians-ICM06 was devoted to this topic.

Recently within our research programme in this field we have undertaken the analysis of fully discrete schemes and discontinuous Galerkin methods, among other topics. In the second half of 2012, a Springer-Briefs volume will be published, in collaboration with A. Marica, with a survey of our contributions in what concerns discontinuous Galerkin methods. An updated presentation of our contributions in this field can be found in the recent 2012 survey paper published in collaboration with S. Ervedoza in the CIME Subseries of Springer. Another Springer Briefs volume on this topic will be published late 2012.

In [22] we have developed a systematic method capable of transferring results on the controllability of time-continuous conservative semigroups into conservative time-discrete versions. This allows to get in a systematic manner convergent fully discrete schemes for the control of waves. In [23] we develop a harmonic analysis method allowing to prove the convergence of the two-grid algorithm for the convergence of numerical methods of control of waves.

As seen above, there are a number of efficient methods for computing accurate approximations of controls for wave-like equations. The situation is different for the heat equation where the strong time-irreversibility makes the problem extremely ill-posed. In [24] we have developed a discrete version of the so-called transmutation method, linking the wave and the heat equation. This allows using the numerical controls of the wave equation to get accurate approximations of the

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[20] E. Z., SIREV, 47 (2) (2005), 197-243 (81 cites ISI-WoK).
[21] S. Ervedoza, E. Z., Numer. Math. 109 (2008), 597—634. (7 citations ISI WoK).
[22] S. Ervedoza, Ch. Zheng, E. Z., J. Functional Analysis, 254 (12) (2008), 3037-3078. (9 citations ISI WoK).
[23] L. Ignat, E. Z., J. European Math. Soc., 11 (2009), 351-391.
[24] A. Münch and E. Z.. J. Inverse Problems, 26(8) 085018 (2010) (6 citations ISI WoK).

controls of the heat equation, bypassing the ill-posedness of the problem.

• **Flow control.** Motivated by joint work with colleagues at AIRBUS Operations and the Spanish Institute of Aerospace Technology (INTA), during the last 7 years we have worked on shape design problems in aeronautics, making various contributions that are currently used by AIRBUS in their designs. In particular, we have developed systematically the continuous approach to optimal shape design and developeda novel application to the complete system of compressible 3D Navier-Stokes equations without using domain integrals in the gradient evaluation [25], 2D Euler equations including the derivation of the Rankine-Hugoniot adjoint internal relations [26], and 3D Reynolds-averaged Navier-Stokes equations with the complete derivation of the continuous adjoint method for the Spalart-Allmaras turbulent model [27]. Most of this original work has been implemented in the TAU code used by Airbus Operations for the design of their commercial aircrafts.

From a theoretical point of view, one of the may concerns is the impact on shape optimization of the shock or shock-like solutions that often arise in the models of fluid mechanics. In a M3AS paper in collaboration with C. Castro and F. Palacios (2008) (8 citations, ISI-WoK), we introduced an alternate direction method that distinguishes and combines the effect of the design parameters on the smooth components of solutions and on the location and nature of its singularities, making descent algorithms much more efficient. We have extended this analysis in various directions, including viscous problems, steady-state models and multi-dimensional ones. We have also shown how the complexity of the boundary functionals to be optimized can be reduced so as to allow numerical methods of lower order to be efficient. This work has been developed mainly in collaboration with F. Palacios, now at Stanford University, which whom we have published four AIAA Journal articles (the most prestigious journal in the aeronautical field, edited by the American Institute of Aeronautics and Astronautics). More recently, in a joint paper with M. Ersoy and E. Feireisl, we have shown that steady state solutions are limits as time tends to infinity of time-evolution ones in the context of one-dimensional scalar conservation laws. We have also developed the corresponding asymptotic analysis for the sensitivity in the presence of shocks. This is a first justification, in a rather simplified setting, of a commonly used reduction in aeronautics when dealing with optimal design problems.

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[25] C. Castro, C. Lozano, F. Palacios, and E. Zuazua "*A Systematic Continuous Adjoint Approach to Viscous Aerodynamic Design on Unstructured Grids*", AIAA Journal Vol. 45, No. 9, pp. 2125-2139, September 2007.

[26] A. Baeza, C. Castro, F. Palacios, and E. Zuazua "*2D Euler shape design on non-regular flows using adjoint Rankine-Hugoniot relations*", AIAA Journal Vol. 47, No. 3, March 2009.

[27] A. Bueno-Orovio, C. Castro, F. Palacios, and E. Zuazua "*Continuous Adjoint Approach for the Spalart-Allmaras Model in Aerodynamic Optimization*", AIAA Journal Vol. 50, No. 3, March 2012.

• **Optimal design.** The theory of optimal design, and, in particular, shape design, has undergone significant progress in the last two decades at the theoretical level, often derived using fine tools of geometric measure theory, showing the existence and regularity of optimal shapes for various problems in elasticity and fluid mechanics. However, much less is known in what concerns the optimal placement of sensors and actuators for control systems modelled by PDE, despite the relevance of these issues in control systems applications. In collaboration with Y. Privat and E. Trélat, we are developing a systematic research agenda in this area. So far we have published two preprints dealing with the 1-d wave equation both for the problem of observability and controllability. We rigorously formulate the problem of searching for the optimal location of observers and controllers with a given volume fraction. By means of a Fourier series decomposition the question is reduced to a spectral problem. Different situations are then considered: Given initial data to be controlled or observed versus the case of uniform observation or control of solutions generated by all possible data, etc. A systematic answer to these problems is given distinguishing three possible scenarios in which the optimal set is the union of a finite number of intervals, a Cantor set, or it does not exist leading to the emergence of relaxation phenomena and optimal densities rather than sets. Our results are the first ones to establish that there are two possible scenarios of increasing complexity (Cantor sets and relaxation) that the numerical simulations of these problems were not able to detect until now. The extension of these results into the multi-dimensional case is work in progress. Note however that in the multi-dimensional case a much more complex behavior is expected since the geometry and energy distribution of the eigenfunctions of the Laplacian is very much related to the dynamics of the billiard of Geometric Optics and its stability properties.

In the broad field of Optimal Design, significant progress has also been made concerning the development of efficient numerical algorithms based on shape differentiation; gradient and level set methods. Despite of this, there is still nowadays an important gap between theory and simulations. Indeed, there are very few examples in which it has been rigorously established that optimal numerical shapes converge, as the mesh-parameter tends to zero, to the optimal shape of the PDE model.

We have developed a systematic research program in this field. In our 2006 paper [28] we have formulated the problem of the convergence of discrete finite element optimal shapes towards the continuous ones in the context of geometric optimal design of Laplace's equation with Dirichlet boundary conditions in 2D and developed a general theory allowing to prove it. This methodology has been recently adapted to the optimal design of conductivity and elasticity coefficients in elliptic problems. In a recent (2011) SIAM J. Multiscale Analysis paper in

[28] D. Chenais & E. Z. JMPA, 85 (2006), 225-249, (6 citations ISI-WoK).

collaboration with J. Casado-Díaz, C. Castro and M. Luna-Laynez, we prove the convergence and analyze the convergence rates of finite-element optimal shapes towards the continuous ones. This is done from a multi-scale point of view since, indeed, the mesh in which the PDE and the shapes are discretized are not necessarily the same. In 1-d we compute the optimal coarseness of the mesh in which the coefficients are discretized with respect to the one in which the PDE is discretized. We also prove that, even if relaxation does not occur because the original continuous design problem has a classical solution, using the relaxed version yields faster convergence rates. This work complements our previous contribution on the numerical analysis of homogenization problems[29] .

[29] R. Orive, E. Z. Multiscale Modeling and Simulation: A SIAM Interdisciplinary Journal, 4 (1) (2005) pp. 36-87(4 citations ISI WoK).