DISEÑO DE EXPERIMENTOS (UN FACTOR)

Modelo: $Y_{ij} \sim N(\mu_i; \sigma^2)$ independientes; $\; i = 1, ..., I; \; j = 1., ..., n_i; \; \sum_i n_i = n.$

$$\bar{y}_{i.} = \frac{1}{n_i} \sum_j y_{ij} \; ; \qquad \bar{y}_{..} = \frac{1}{n} \sum_i n_i \bar{y}_{i.}$$

$$\hat{\mu}_i = \bar{y}_{i.} = \frac{1}{n_i} \sum_j y_{ij} \, , \; i = 1, ..., I \qquad \hat{\sigma}^2 = S_R^2 = \frac{1}{n-I} \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2$$

$$IC_{1-\alpha}(\mu_i) = \left(\bar{y}_{i.} \pm t_{n-I;\alpha/2} \, S_R \sqrt{\frac{1}{n_i}} \right) \qquad IC_{1-\alpha}(\sigma^2) = \left(\frac{(n-I)S_R^2}{\chi^2_{n-I;\alpha/2}} \; ; \frac{(n-I)S_R^2}{\chi^2_{n-I;1-\alpha/2}} \right)$$

Tabla ANOVA

Suma de cuadrados	g.l.	Varianza	Estadístico
$SCE = \sum_i n_i(\bar{y}_{i.} - \bar{y}_{..})^2$	$I-1$	$\frac{SCE}{I-1}$	$F = \frac{SCE/(I-1)}{SCR/(n-I)}$
$SCR = \sum_i \sum_j (y_{ij} - \bar{y}_{i.})^2$	$n-I$	$S_R^2 = \frac{SCR}{n-I}$	
$SCT = \sum_i \sum_j (y_{ij} - \bar{y}_{..})^2$	$n-1$		

$$IC_{1-\alpha}(\mu_i - \mu_j) = \left(\bar{y}_{i.} - \bar{y}_{j.} \pm t_{n-I;\alpha/2} \, S_R \sqrt{\frac{1}{n_i} + \frac{1}{n_j}} \right) \qquad ; \qquad S_R^2 = \frac{\sum_i (n_i - 1)s_i^2}{n-I}$$

DISEÑO DE EXPERIMENTOS (DOS FACTORES SIN INTERACCIÓN)

$Y_{ij} \sim N(\mu + \alpha_i + \beta_j; \sigma^2)$ independientes; $\; i = 1, \dots, I; \; j = 1, \dots, J$

$$\hat{\mu} = \bar{y}_{..} = \frac{1}{IJ} \sum_i \sum_j y_{ij}$$

$$\hat{\alpha}_i = \bar{y}_{i.} - \bar{y}_{..} = \frac{1}{J} \sum_j y_{ij} - \bar{y}_{..}$$

$$\hat{\beta}_j = \bar{y}_{.j} - \bar{y}_{..} = \frac{1}{I} \sum_i y_{ij} - \bar{y}_{..}$$

$$\hat{\sigma}^2 = S_R^2 = \frac{1}{(I-1)(J-1)} \sum_i \sum_j (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 = \frac{1}{(I-1)(J-1)} \sum_i \sum_j \hat{e}_{ij}^2$$

Tabla ANOVA

Suma de cuadrados	g.l.	Varianza	Estadístico
$SCE(\alpha) = J \sum_i \hat{\alpha}_i^2$	$I-1$	$\frac{SCE(\alpha)}{I-1}$	$F(\alpha)$
$SCE(\beta) = I \sum_j \hat{\beta}_i^2$	$J-1$	$\frac{SCE(\beta)}{J-1}$	$F(\beta)$
$SCR = \sum_i \sum_j \hat{e}_{ij}^2$	$(I-1)(J-1)$	$\frac{SCR}{(I-1)(J-1)}$	
$SCT = \sum_i \sum_j (y_{ij} - \bar{y}_{..})^2$	$IJ-1$		

siendo: $\qquad F(\alpha) = \dfrac{SCE(\alpha)/(I-1)}{SCR/(I-1)(J-1)} \; ; \qquad F(\beta) = \dfrac{SCE(\beta)/(J-1)}{SCR/(I-1)(J-1)}$

$$IC_{1-\alpha}(\alpha_i - \alpha_j) = \left(\bar{y}_{i.} - \bar{y}_{j.} \pm t_{(I-1)(J-1);\alpha/2} \, S_R \sqrt{\frac{1}{J} + \frac{1}{J}} \right)$$

$$IC_{1-\alpha}(\beta_i - \beta_j) = \left(\bar{y}_{.i} - \bar{y}_{.j} \pm t_{(I-1)(J-1);\alpha/2} \, S_R \sqrt{\frac{1}{I} + \frac{1}{I}} \right)$$

DISEÑO DE EXPERIMENTOS (DOS FACTORES CON INTERACCIÓN)

$Y_{ijk} \sim N(\mu + \alpha_i + \beta_j + (\alpha\beta)_{ij}; \sigma^2)$ independientes; $i = 1, ..., I$; $j = 1., ..., J$; $k = 1, ..., K$

$$
\begin{aligned}
\hat{\mu} &= \bar{y}_{...} = \frac{1}{IJK} \sum_i \sum_j \sum_k y_{ijk} \\
\hat{\alpha}_i &= \bar{y}_{i..} - \bar{y}_{...} = \frac{1}{JK} \sum_j \sum_k y_{ijk} - \bar{y}_{...} \\
\hat{\beta}_j &= \bar{y}_{.j.} - \bar{y}_{...} = \frac{1}{IK} \sum_i \sum_k y_{ijk} - \bar{y}_{...} \\
\hat{(\alpha\beta)}_{ij} &= \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...} = \frac{1}{K} \sum_k y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...} \\
\hat{\sigma}^2 &= S_R^2 = \frac{1}{IJ(K-1)} \sum_i \sum_j \sum_k (y_{ijk} - \bar{y}_{ij.})^2
\end{aligned}
$$

Tabla ANOVA

Suma de cuadrados	G.l.	Varianza	F
$SCE(\alpha) = JK \sum_i \hat{\alpha}_i^2$	$I-1$	$\frac{SCE(\alpha)}{I-1}$	$F(\alpha)$
$SCE(\beta) = IK \sum_j \hat{\beta}_j^2$	$J-1$	$\frac{SCE(\beta)}{J-1}$	$F(\beta)$
$SCE(\alpha\beta) = K \sum_i \sum_j (\hat{\alpha\beta})_{ij}^2$	$(I-1)(J-1)$	$\frac{SCE(\alpha\beta)}{(I-1)(J-1)}$	$F(\alpha\beta)$
$SCR = \sum_i \sum_j \sum_k (y_{ijk} - \bar{y}_{ij.})^2$	$IJ(K-1)$	$\frac{SCR}{IJ(K-1)}$	
$SCT = \sum_i \sum_j \sum_k (y_{ijk} - \bar{y}_{...})^2$	$IJK-1$		

siendo los estadísticos:

$$
F(\alpha) = \frac{SCE(\alpha)/(I-1)}{SCR/IJ(K-1)} \quad F(\beta) = \frac{SCE(\beta)/(J-1)}{SCR/IJ(K-1)} \quad F(\alpha\beta) = \frac{SCE(\alpha\beta)/(I-1)(J-1)}{SCR/IJ(K-1)}
$$

REGRESIÓN LINEAL SIMPLE

Modelo: $Y_i \sim N(\beta_0 + \beta_1 x_i; \sigma^2)$ independientes, $i = 1, ..., n$.

$$
\begin{aligned}
\hat{\beta}_1 &= \frac{cov}{v_x} \\
\hat{\beta}_0 &= \bar{y} - \frac{cov}{v_x} \bar{x} = \bar{y} - \hat{\beta}_1 \bar{x} \\
\hat{\sigma}^2 &= S_R^2 = \frac{1}{n-2} \sum_i (y_i - \hat{y}_i)^2 = \frac{1}{n-2} \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2
\end{aligned}
$$

$$
\begin{aligned}
IC_{1-\alpha}(\beta_0) &= \left(\hat{\beta}_0 \pm t_{n-2;\alpha/2} \, S_R \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{nv_x}} \right) \\
IC_{1-\alpha}(\beta_1) &= \left(\hat{\beta}_1 \pm t_{n-2;\alpha/2} \, S_R \sqrt{\frac{1}{nv_x}} \right) \\
IC_{1-\alpha}(\sigma^2) &= \left(\frac{(n-2)S_R^2}{\chi_{n-2;\alpha/2}^2} \, ; \frac{(n-2)S_R^2}{\chi_{n-2;1-\alpha/2}^2} \right)
\end{aligned}
$$

Tabla ANOVA			
Suma de cuadrados	G.l.	Varianza	Estadístico
$SCE = \sum_i (\hat{y}_i - \bar{y})^2$	1	$\frac{SCE}{1}$	$F = \frac{SCE/1}{SCR/(n-2)}$
$SCR = \sum_i (y_i - \hat{y}_i)^2$	$n-2$	$\frac{SCR}{n-2}$	
$SCT = \sum_i (y_i - \bar{y})^2$	$n-1$		

$$SCE = n v_y r^2 \; ; \qquad SCR = n v_y (1 - r^2) \; ; \qquad \text{donde } r = \frac{cov}{\sqrt{v_x v_y}}$$

$$IC_{1-\alpha}(\text{valor medio de } Y) = \left(\hat{y}_0 \pm t_{n-2;\alpha/2} \; S_R \sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{n v_x}} \right)$$

$$IC_{1-\alpha}(\text{valor individual de } Y) = \left(\hat{y}_0 \pm t_{n-2;\alpha/2} \; S_R \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{n v_x}} \right)$$

REGRESIÓN LINEAL MÚLTIPLE

Modelo: $Y_i \sim N(\beta_0 + \beta_1 x_{1i} + ... + \beta_k x_{ki}; \sigma^2)$ independientes, $i = 1, ..., n$.

$$\hat{\sigma}^2 = S_R^2 = \frac{1}{n-k-1} \sum_i (y_i - \hat{y}_i)^2 = \frac{1}{n-k-1} \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{1i} - \cdots - \hat{\beta}_k x_{ki})^2$$

$$IC_{1-\alpha}(\beta_i) = \left(\hat{\beta}_i \pm t_{n-k-1;\alpha/2}(\text{error típico de } \hat{\beta}_i) \right), \; i = 0, \ldots, k$$

$$H_0 : \beta_i \leq 0 \;\; \Rightarrow \;\; \mathcal{R} = \left\{ \frac{\hat{\beta}_i}{\text{error típico de } \hat{\beta}_i} > t_{n-k-1;\alpha} \right\}$$

$$H_0 : \beta_i \geq 0 \;\; \Rightarrow \;\; \mathcal{R} = \left\{ \frac{\hat{\beta}_i}{\text{error típico de } \hat{\beta}_i} < t_{n-k-1;1-\alpha} \right\}$$

Tabla ANOVA			
Suma de cuadrados	G.l.	Varianza	Estadístico
$SCE = \sum_i (\hat{y}_i - \bar{y})^2$	k	$\frac{SCE}{k}$	$F = \frac{SCE/k}{SCR/(n-k-1)}$
$SCR = \sum_i (y_i - \hat{y}_i)^2$	$n-k-1$	$\frac{SCR}{n-k-1}$	
$SCT = \sum_i (y_i - \bar{y})^2$	$n-1$		

$$R^2 = \frac{SCE}{SCT} = \frac{SCT - SCR}{SCT} \; ; \qquad F = \frac{R^2}{1 - R^2} \frac{n-k-1}{k}$$