Recent developments in nonlocal transport equations with nonlinear diffusion

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Nonlocal transport PDEs have gained a great interest lately in tha applied mathematics community, both for their strong connection with singular aggregation phenomena "a la" Keller-Segel and for the complexity in their asymptotic behaviour depending on some initial parameters. I will focus on a specific case, namely that of an aggregative nonlocal force possibly coupled with a quadratic diffusion. The latter arises as a large particle limit of a short range repulsive force under certain conditions. Without diffusion, the solution tends to concentrate in infinite or finite time depending on the behaviour of the attractive kernel at zero. In many dimensions, one can carry out a robust existence theory which allows to deal with concentrated solutions as well (see part of the course by J. A. Carrillo). The effect of quadratic diffusion is that of producing a complex behaviour, namely existence of stationary profiles vs, large time decay. This depends on the total mass of the interaction kernel. In one space dimension the stationary state can be proven to be unique up to translation and mass conservation. This fact is somewhat surprising as the model can be seen as gradient flow of a non convex functional. The strategy of the uniqueness proof relies on a "strong" version of the Krein-Rutman theorem.

The parabolic fractional obstacle problem

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In recent years, there has been an increasing interest in studying constrained variational problems with a fractional diffusion. One of the motivations comes from mathematical finance: jump-diffusion processes where incorporated into the theory of option evaluation to introduce discontinuous paths in the dynamics of the stock's prices in order to take into account large price changes, and they have become increasingly popular for modeling market fluctuations, both for risk management and option pricing purposes.

In a joint paper with Luis Caffarelli we study the parabolic version of the fractional obstacle problem, i.e. where the elliptic part of the operator is given (at least at the leading order) by a fractional Laplacian. We prove optimal spatial regularity and almost optimal time regularity of the solution, recovering in particular the optimal regularity result of Caffarelli-Salsa-Silvestre for the stationary case. To obtain this result, we crucially exploit the fact that the solution coincides with the obstacle at the initial time, which corresponds to the fact that (for the backward operator) the stock's price coincides with the payoff at the final time.

A fundamental functional inequality on the 2-dimensional sphere and its ramifications

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I will describe how a refinement of the two-dimensional Sobolev inequality is closely related to the problem of prescribing Gaussian curvature on the sphere, to the optimization of determinants of Laplacians within a conformal class, to the Landau solutions of Navier-Stokes equations in 3-dimension, as well as to certain Liouville equations arising in quantum field theory. I will also present a joint work with Chang-Shou Lin, in which a partial solution is given to a question of A. Chang and P. Yang about the best constant in the energy inequality in question.

Nonlinear Integrate and Fire Neuron Models

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Nonlinear Noisy Leaky Integrate and Fire models for neurons networks can be written as Fokker-Planck equations on the probability density of neurons. We show global existence and uniqueness of the solution for inhibitory networks, together with some comments on asymptotic decay to equilibrium. This is joint work with J. Carrillo, M. Gualdani and M. Schonbek.

Nonlinear waves and free boundary problems

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I will first describe a free boundary problem where the positivity phase lies on a codimension 1 set. I will describe the optimal regularity of the solution, its non degeneracy close to the free boundary and the regularity of the free boundary. This is joint work with L. Caffarelli and J.-M. Roquejoffre. Then I will use these results to produce a non linear wave attached to a boundary reaction problem. This is a non local model and I will provide also some asymptotic results.