Local bounds and Harnack inequalities for very fast evolution equations

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We investigate qualitative properties of local solutions $u(t,x) \geq 0$ to the fast diffusion equation, $\partial_t u = \Delta(u^m)/m$ with $m < 1$, and $\partial_t u = \Delta_p u = \nabla \cdot (|\nabla u|^{p-2}\nabla u)$ with $1 < p < 2$, corresponding to general nonnegative initial data. Our main results are quantitative positivity and boundedness estimates for locally defined solutions in domains of $[0,T] \times \mathbb{R}^d$. They combine into forms of new Harnack inequalities that are typical of fast diffusion equations. Such results are new for m and p in the so-called very fast diffusion range, precisely for all $m \leq m_c = (d-2)/d$, and/or for all $1 < p \leq 2d/(d+1)$, where d is the dimension of the Euclidean space $\mathbb{R}^d$. In the supercritical case we recover the (sharp) results existing in literature with a different proof. The boundedness results are true even for $m \leq 0$, or $p = 1$, while the positivity ones cannot be true in that range.

For the fast p-Laplacian we also prove a new local energy inequality for suitable norms of the gradients of the solutions, which can be extended to more general operators of p-Laplacian type. As a consequence, we show that bounded local weak solutions are indeed local strong solutions for any $1 < p < 2$, more precisely $\partial_t u \in L^2_{\text{loc}}$.

This is a joint work with J. L. Vázquez.

How big can be a hole of a thin film?

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The thickness u of a thin film can be modeled as a solution of a PDE. In this talk I will study the maximal size of the rupture set $[u = 0]$ or, more precisely, the maximal dimension of this set. Depending on a parameter, it is possible to avoid the formation of free boundaries.

On the approximation of orientation-preserving homeomorphisms by smooth or piecewise affine ones

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In the context of non-linear elasticity, it would be of primary importance to approximate $W^{1,2}$ orientation-preserving homeomorphisms via smooth or piecewise affine ones. Unfortunately, this is not easily achievable via the standard mollification or other smoothing techniques. We will describe the history and the state-of-the-art in this problem, and we will show some very new results.

Nonlinear fractional diffusion

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We develop a theory of existence and uniqueness for a nonlinear diffusion model involving fractional powers of the Laplacian. We devote special attention to nonnegative solutions, which are proved to be continuous and positive for all positive times. The continuous dependence of solutions on the involved parameters and on the initial data is also considered.

This is a joint work with Arturo de Pablo, Ana Rodríguez and Juan Luis Vázquez.

Singularity evolution in very fast diffusion equations

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It is known that diffusive systems may spontaneously develop singularities, and many facets of such processes have been described well by mathematical theory. The talk addresses a related aspect that appears far from being completely understood: *What happens* beyond *the onset of a singularity?* In order to concentrate here on the interplay between the singularity and *diffusion* (and not reaction, for instance), the focus will be on the pure – possibly nonlinear – diffusion equation $u_t = \Delta u^m$, $m > 0$, without source terms, in a bounded domain $\Omega \subset \mathbb{R}^n$. The initial data are supposed to possess an isolated singularity of type $u_0(x) \simeq |x|^{-\gamma}$, $x \simeq 0 \in \Omega$, $\gamma > 0$.

It turns out that when $n \geq 3$ and $m < m_c := \frac{n-2}{n}$, various types of behavior occur: Depending on the size of γ, the singularity may either disappear immediately, or remain for a while and then disappear, or stay for all positive times. In the latter case, the singularity can either force the solution to blow up everywhere as $t \to \infty$, or remain essentially unchanged in strength for all times, or disappear at least in the limit $t \to \infty$.

This is joint work with Juan Luis Vázquez.