Regularity and optimality conditions for a free boundary problem

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I present some recent results on the equilibrium configurations of a variational model for the epitaxial growth of a thin film with planar symmetries on a thick substrate obtained in colllaboration with M.Morini. In our model the total energy is given by the sum of an elastic energy and a surface term of the type

$$\int_{\Omega_\Gamma} W(E(u))dx + \sigma_f \mathcal{H}^1(\Gamma)\,,$$

where Ω_Γ is the region occupied by the film (bounded from above by the graph Γ), W is a quadratic form depending on the symmetric part $E(u)$ of the gradient of the displacement u and $\mathcal{H}^1$ denotes the 1-dimensional Hausdorff measure.
Though many properties have been well understood since long by means of numerical or asintotic expansion arguments, rigorous analytical proofs of the existence and regularity of minimal configurations have been obtained only in the last few years.
In the lectures I will discuss a second variation approach that led us to determine analytically the critical volume thresholds for the local and global minimality of the flat configuration and to understand under which conditions cusp singularities do not form, once the flat configuration becomes unstable.

[BC] Bonnetier E. & Chambolle A., *Computing the equilibrium configuration of epitaxially strained crystalline films,* SIAM J. Appl. Math. **62** (2002), 1093–1121

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[FM] Fusco N. & Morini M., *Equilibrium configurations of epitaxially strained elastic films: qualitative properties of solutions,* to appear

[ST] Spencer B. J. & Tersoff J., *Equilibrium shapes and properties of epitaxially strained islands* Physical Review Letters **79** (1997), 4858-4861.

New challenges in Kinetic and Quantum equations

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We consider a class of phase space measures, which naturally arise in the Bohmian interpretation of quantum mechanics (when written in a Lagrangian form). We study the so-called classical limit of these Bohmian measures, in dependence on the scale of oscillations and concentrations of the sequence of wave functions under consideration. The obtained results are consequently compared to those derived via semi-classical Wigner measures. To this end, we shall also give a connection to the theory of Young measures and prove several new results on Wigner measures themselves. We believe that our analysis sheds new light on the classical limit of Bohmian quantum mechanics and gives further insight on oscillation and concentration effects of semi-classical wave functions.

Differential Equations of Mathematical Biology
(Adaptive evolution and concentrations in parabolic PDEs)

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Living systems are subject to constant evolution through the three processes of population growth, selection and mutations, a principle discovered by C. Darwin. In a very simple, general and idealized description, their environment can be considered as a nutrient shared by all the population. This allows certain individuals, characterized by a 'phenotypical trait', to expand faster because they are better adapted to use the environment. This leads to select the 'fittest trait' in the population (singular point of the system). On the other hand, the new-born individuals undergo small variation of the trait under the effect of genetic mutations. In these circumstances, is it possible to describe the dynamical evolution of the current trait? A new area of population biology that aims at describing mathematically these processes is born in the 1980's under the name of 'adaptive dynamics' and, compared to population genetics, considers usually asexual reproduction, a continuous phenotypical trait and population growth.

We will give a self-contained mathematical model of such dynamics, based on parabolic equations, and show that an asymptotic method allows us to formalize precisely the concepts of monomorphic or polymorphic population. Then, we can describe the evolution of the 'fittest trait' and eventually to compute various forms of branching points which represent the cohabitation of two different populations.

The concepts are based on the asymptotic analysis of the above mentioned parabolic equations once appropriately rescaled. This leads to concentrations of the solutions and the difficulty is to evaluate the weight and position of the moving Dirac masses that desribe the population. We will show that a new type of Hamilton-Jacobi equation, with constraints, naturally describes this asymptotic. Some additional theoretical questions as uniqueness for the limiting H.-J. equation will also be addressed.

This course is based on collaborations with G. Barles, J. Carrillo, S. Cuadrado, O. Diekmann, M. Gauduchon, S. Genieys, P.-E. Jabin, S. Mirahimmi, S. Mischler and P. E. Souganidis.

Theory of diffusion. Nonlinear models and applications

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In these four talks we will describe the theory of the nonlinear heat flows known under the label of Nonlinear Diffusion. The main subject is nonlinear PDEs, but will pause to consider relevant connections with linear PDEs, Functional Analysis, ODEs, Geometry and Physics.

After introducing the general area and some of its main topics, we will focus on the theory of porous medium flows as a paradigm. We will describe the optimal existence theory based on peculiar estimates and the typical features, like finite propagation, free boundaries and extinction in finite time.

We will also stress the strong connection with the theory of nonlinear elliptic equations, in particular equations of the Fokker-Planck form and reaction-diffusion form. A main area of research consists of the study of the long-time behaviour of heat flows.

Stability questions for solutions of elliptic equations relate to interesting questions for the convergence of parabolic flows, and this also leads to functional inequalities of Hardy-Poincaré type. Geometrical questions are very relevant now and some will be presented, like connections with Ricci and Yamabe flows.

A quite novel feature currently under investigation is the theory of nonlinear diffusion where the Laplace operator is replaced by fractional Laplacian operators of different types.