The role of Wolff potentials in the analysis of degenerate parabolic equations

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September 19, 2011

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Part 1: Ellipticity

The classical potential estimates

- **Consider the model case**

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The classical potential estimates

- **Consider the model case**

$$-\triangle u = \mu \qquad \text{in } \mathbb{R}^n$$

- **We have**

$$u(x) = \int G(x,y)\mu(y)\, dy$$

where

$$G(x,y) \approx \begin{cases} |x-y|^{2-n} & \text{se } n > 2 \\ -\log|x-y| & \text{se } n = 2 \end{cases}$$

Estimates via Riesz potentials

- Previous formula gives

$$|u(x)| \lesssim \int_{\mathbb{R}^n} \frac{d|\mu|(y)}{|x-y|^{n-2}} = I_2(|\mu|)(x)$$

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- while, after differentiation, we obtain

$$|Du(x)| \lesssim \int_{\mathbb{R}^n} \frac{d|\mu|(y)}{|x-y|^{n-1}} = I_1(|\mu|)(x)$$

What happens in the nonlinear case?

- **For instance for nonlinear equations with linear growth**

$$-\text{div } a(Du) = \mu$$

that is equations well posed in $W^{1,2}$ (p-growth and $p = 2$)

- **And degenerate ones like**

$$-\text{div } (|Du|^{p-2}Du) = \mu$$

Local versions

- **In bounded domains one uses**

$$\mathbf{I}^{\mu}_{\beta}(x,R) := \int_0^R \frac{\mu(B(x,\varrho))}{\varrho^{n-\beta}}\,\frac{d\varrho}{\varrho} \qquad\qquad \beta \in (0,n]$$

since

$$\begin{aligned}\mathbf{I}^{\mu}_{\beta}(x,R) &\lesssim \int_{B_R(x)} \frac{d\mu(y)}{|x-y|^{n-\beta}} \\ &= I_\beta(\mu \llcorner B(x,R))(x) \\ &\leq I_\beta(\mu)(x)\end{aligned}$$

for non-negative measures

The setting

- **We consider equations**

$$-\text{div } a(Du) = \mu$$

- **under the assumptions**

$$\begin{cases} |a(z)| + |\partial a(z)||z| \leq L|z|^{p-1} \\ \nu^{-1}|z|^{p-2}|\lambda|^2 \leq \langle \partial a(x,z)\lambda, \lambda \rangle \end{cases}$$

with

$$p \geq 2$$

this last bound is assumed in order to keep the exposition brief

Non-linear potentials

- **The nonlinear Wolff potential is defined by**

$$\mathbf{W}^{\mu}_{\beta,p}(x,R) := \int_0^R \left(\frac{|\mu|(B(x,\varrho))}{\varrho^{n-\beta p}}\right)^{\frac{1}{p-1}} \frac{d\varrho}{\varrho} \qquad \beta \in (0, n/p]$$

which for $p = 2$ reduces to the usual Riesz potential

$$\mathbf{I}^{\mu}_{\beta}(x,R) := \int_0^R \frac{\mu(B(x,\varrho))}{\varrho^{n-\beta}} \frac{d\varrho}{\varrho} \qquad \beta \in (0, n]$$

- **The nonlinear Wolff potential** plays in nonlinear potential theory the same role the Riesz potential plays in the linear one

A fundamental estimate

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Another approach to this result has been given by Trudinger & Wang (Amer. J. Math. 02)

Integral estimates follow via Wolff inequalities

- **We have**

$$\mu \in L^q \Longrightarrow \mathbf{W}^{\mu}_{\beta,p} \in L^{\frac{nq(p-1)}{n-qp\beta}} \qquad q \in (1,n)$$

with related explicit estimates, also in Marcinkiewicz spaces

- **Such a property allows to reduce the study of integrability of solutions to that of nonlinear potentials**
- **The key is the following inequality:**

$$\int_0^\infty \left(\frac{|\mu|(B(x,\varrho))}{\varrho^{n-\beta p}}\right)^{\frac{1}{p-1}} \frac{d\varrho}{\varrho} \lesssim I_\beta \left\{[I_\beta(|\mu|)]^{\frac{1}{p-1}}\right\}(x)$$

the last quantity is called Havin-Maz'ja potential

The potential gradient estimate for $p = 2$

Theorem (Min., JEMS 2011)

$$|D_\xi u(x)| \lesssim \mathbf{I}_1^{|\mu|}(x,R) + \fint_{B(x,R)} |D_\xi u| \, dy$$

holds for almost every point x and $\xi \in \{1, \ldots, n\}$

A general potential gradient estimate

Theorem (Duzaar & Min., Amer. J. Math. 2011)

$$|Du(x)| \lesssim \mathbf{W}^{\mu}_{1/p,p}(x,R) + \fint_{B(x,R)} |Du|\, dy$$

holds for almost every point x

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holds for almost every point x

This means

$$|Du(x)| \leq c \int_0^R \left(\frac{|\mu|(B(x,\varrho))}{\varrho^{n-1}} \right)^{\frac{1}{p-1}} \frac{d\varrho}{\varrho} + c \fint_{B(x,R)} |Du|\, dx$$

General potential gradient estimate

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$$|Du(x)| \lesssim \mathbf{W}^{\mu}_{1/p,p}(x,R) + \fint_{B(x,R)} |Du|\, dy$$

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First remarks

- **Since**

$$\mu \in L^q \Longrightarrow \mathbf{W}^{\mu}_{1/p,p} \in L^{\frac{nq(p-1)}{n-q}} \qquad q \in (1,n)$$

therefore, for instance

$$\mu \in L^q \Longrightarrow Du \in L^{\frac{nq(p-1)}{n-q}}_{\mathrm{loc}}(\Omega) \qquad q \in (1,n)$$

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- More in general, estimates in all rearrangement invariant spaces follow, recovering all those already known for the model case $\mathrm{div}\,(|Du|^{p-2}Du) = \mu$, and fixing **open borderline cases**

The two estimates

- **The two potential estimates are**

$$|u(x)| \lesssim \mathbf{W}^{\mu}_{1,p}(x,R) + c \fint_{B(x,R)} |u|\, dy$$

and

$$|Du(x)| \leq c\mathbf{W}^{\mu}_{1/p,p}(x,R) + \fint_{B(x,R)} |Du|\, dy$$

- **They basically provide size estimates on u and Du**
- **The aim is now to provide estimates on the oscillations of solutions and/or alternatively, on intermediate derivatives**

Calderón spaces of DeVore & Sharpley

- **The following definition is due to DeVore & Sharpley (Mem. AMS, 1982)**
- Let $\alpha \in (0,1]$, $q \geq 1$, and let $\Omega \subset \mathbb{R}^n$ be a bounded open subset. A measurable function v, finite a.e. in Ω, belongs to the Calderón space $C^{\alpha}_{q}(\Omega)$ if and only if there exists a nonnegative function $m \in L^q(\Omega)$ such that

$$|v(x) - v(y)| \leq [m(x) + m(y)]|x - y|^{\alpha}$$

holds for almost every couple $(x, y) \in \Omega \times \Omega$.

Calderón spaces of DeVore & Sharpley

- In other words

$$m(x) \approx \partial^{\alpha} v(x)$$

- Indeed DeVore & Sharpley take

$$M^{\alpha}_{\#} v(x) = \sup_{B(x,\varrho)} \varrho^{-\alpha} \fint_{B(x,\varrho)} |v(y) - (v)_{B(x,\varrho)}| \, dy$$

- For $\alpha \in (0,1)$ and $q > 1$ we have

$$W^{\alpha,q} \subset C^{\alpha,q} \subset W^{\alpha-\varepsilon,q}$$

therefore such spaces, although not being of interpolation type, are just another way to say "fractional differentiabilty"

A "universal potential estimate"

Theorem (Kuusi & Min.)

The estimate

$$
\begin{aligned}
&|u(x)-u(y)| \\
&\leq c\left[\mathbf{W}^{\mu}_{1-\frac{\alpha(p-1)}{p},p}(x,R)+\mathbf{W}^{\mu}_{1-\frac{\alpha(p-1)}{p},p}(y,R)\right]|x-y|^{\alpha} \\
&\qquad +c \fint_{B_R}|u|\,d\xi\cdot\left(\frac{|x-y|}{R}\right)^{\alpha}
\end{aligned}
$$

holds uniformly in $\alpha \in [0,1]$, whenever $x,y \in B_{R/4}$

- The cases $\alpha = 0$ and $\alpha = 1$ give back the two known potential estimates as endpoint cases

The homogeneous case

- The estimate tells that

$$“\partial^{\alpha} u(x) \lesssim \mathbf{W}^{\mu}_{1-\frac{\alpha(p-1)}{p},p}(x,R)”$$

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- The case $\mu = 0$ reduces to the classical estimate

$$|u(x) - u(y)| \leq \fint_{B_R} |u| \, d\xi \cdot \left(\frac{|x-y|}{R} \right)^{\alpha}$$

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$$|u(x) - u(y)| \leq \fint_{B_R} |u| \, d\xi \cdot \left(\frac{|x-y|}{R} \right)^\alpha$$

- In the case $p = 2$ we have

$$|u(x) - u(y)| \leq c \left[\mathbf{I}^{|\mu|}_{2-\alpha}(x,R) + \mathbf{I}^{|\mu|}_{2-\alpha}(y,R) \right] |x-y|^\alpha$$
$$+ c \fint_{B_R} |u| \, d\xi \cdot \left(\frac{|x-y|}{R} \right)^\alpha$$

which in the classical case $-\triangle u = \mu$ can be derived directly from the standard representation formula via potentials

The second universal potential estimate

Theorem (Kuusi & Min.)

The estimate

$$|Du(x) - Du(y)|$$
$$\leq c \left[\mathbf{W}^{\mu}_{1-\frac{(1+\alpha)(p-1)}{p},p}(x,R) + \mathbf{W}^{\mu}_{1-\frac{(1+\alpha)(p-1)}{p},p}(y,R) \right] |x-y|^{\alpha}$$
$$+ c \fint_{B_R} |Du| \, d\xi \cdot \left(\frac{|x-y|}{R} \right)^{\alpha}$$

holds whenever $\alpha < \alpha_M$, *whenever* $x, y \in B_{R/4}$

- The case $\alpha = 0$ gives back the gradient potential estimate

Part 2: Parabolicity

Parabolics

- **The model case is here given by**

$$u_t - \operatorname{div}(|Du|^{p-2}Du) = \mu\,,$$

more in general we consider

$$u_t - \operatorname{div} a(Du) = \mu\,.$$

- A basic reference for existence and a priori estimates is the work of Boccado, Dall'Aglio, Galloüet and Orsina, J. Funct. Anal., 1997

Problematic aspects

- For basic scaling reasons the previous potential estimates do not hold in the case $p \neq 2$
- For the case $p = 2$ it holds

Theorem (Duzaar & Min., Amer. J. Math. 2011)

$$|Du(x,t)| \lesssim \mathbf{I}_1^{|\mu|}(x,t;r) + \fint_{Q_r(x,t)} |Du| \, dz$$

holds for almost every point (x,t) *and* $\xi \in \{1,\ldots,n\}$

Problematic aspects

- Here $\mathbf{I}_1^{|\mu|}(x, t; r)$ denotes the parabolic Riesz potential

$$\mathbf{I}_\beta^{|\mu|}(x, t; r) := \int_0^r \frac{|\mu|(Q_\varrho(x, t))}{\varrho^{N-\beta}} \frac{d\varrho}{\varrho}, \quad \beta < N := n + 2$$

  and

$$Q_\varrho(x, t) = B(x, \varrho) \times (t - \varrho^2, t)$$

  is a standard parabolic cylinder
- **The case $p \neq 2$ is a very different story**

Recall the elliptic estimate

Theorem (Duzaar & Min., Amer. J. Math. 2011)

$$|Du(x)| \lesssim \mathbf{W}^{\mu}_{1/p,p}(x,R) + \fint_{B(x,R)} |Du|\, dy$$

holds for almost every point x

Recall the elliptic estimate

Theorem (Duzaar & Min., Amer. J. Math. 2011)

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holds for almost every point x

This means

$$|Du(x)| \lesssim \int_0^R \left(\frac{|\mu|(B(x,\varrho))}{\varrho^{n-1}}\right)^{\frac{1}{p-1}} \frac{d\varrho}{\varrho} + \fint_{B(x,R)} |Du| \, dx$$

Another way to say the same thing

if

$$c \fint_{B(x,R)} |Du| \, dy + c\mathbf{W}^{\mu}_{1/p,p}(x, R) \leq \lambda$$

then

$$|Du(x)| \leq \lambda$$

The intrinsic geometry of DiBenedetto

- **The basic analysis is the following: consider intrinsic cylinders**

$$Q_\varrho^\lambda(x,t) = B(x,\varrho) \times (t - \lambda^{2-p}\varrho^2, t)$$

where it happens that

$$|Du| \approx \lambda \qquad \text{in } Q_\varrho^\lambda(x,t)$$

then the equation behaves as

$$u_t - \lambda^{p-2}\triangle u = 0$$

that is, scaling back in the same cylinder, as the heat equation

- **On intrinsic cylinders estimates "ellipticize"; in particular, they become homogeneous**

DiBenedetto's intrinsic estimate

- The effect of intrinsic geometry

Theorem (DiBenedetto & Friedman, Crelle J. 85)

There exists a universal constant $c \geq 1$ such that

$$c \left(\fint_{Q_r^\lambda(x,t)} |Du|^{p-1} \, dz \right)^{1/(p-1)} \leq \lambda$$

then

$$|Du(x,t)| \leq \lambda$$

Intrinsic Wolff potentials

- **Define the intrinsic Wolff potential** such that

$$\mathbf{W}_{\lambda}^{\mu}(x,t;r) := \int_0^r \left[\frac{|\mu|(Q_{\varrho}^{\lambda}(x,t))}{\lambda^{2-p}\varrho^{N-1}} \right]^{1/(p-1)} \frac{d\varrho}{\varrho}$$

- **Note that**

$$\mathbf{W}_{\lambda}^{\mu}(x,t;r) = \mathbf{I}_{1}^{|\mu|}(x,t;r) \qquad \text{when } p = 2$$

and

$$\mathbf{W}_{\lambda}^{\mu}(x,t;r) = \mathbf{W}_{1/p,p}^{\mu}(x,r) \qquad \text{when } \mu \text{ is time independent}$$

this is the elliptic case

The parabolic Wolff gradient bound

Theorem (Kuusi & Min.)

There exists a universal constant $c \geq 1$ such that

$$c\mathbf{W}^{\mu}_{\lambda}(x,t;r) + c\left(\fint_{Q_r^{\lambda}(x,t)} |Du|^{p-1}\,dz\right)^{1/p-1} \leq \lambda$$

then

$$|Du(x,t)| \leq \lambda$$

- **When $\mu \equiv 0$ this reduces to the sup estimate of DiBenedetto & Friedman (Crelles J. 84)**

Sharpness

- Consider the equation

$$u_t - \operatorname{div}\left(|Du|^{p-2}Du\right) = \delta,$$

where δ denotes the Dircac unit mass charging the origin

- The so called Barenblatt (fundamental solution) is

$$\mathcal{B}_p(x,t) = \begin{cases} t^{-\frac{n}{\theta}}\left(c_b - \theta^{\frac{1}{1-p}}\left(\frac{p-2}{p}\right)\left(\frac{|x|}{t^{1/\theta}}\right)^{\frac{p}{p-1}}\right)_+^{\frac{p-1}{p-2}} & t > 0 \\ 0 & t \leq 0\,. \end{cases}$$

for $\theta = n(p-2) + p$ and a suitable constant c_b such that

$$\int_{\mathbb{R}^n} \mathcal{B}_p(x,t)\,dx = 1 \qquad \forall\ t > 0$$

Sharpness

- A direct computation shows the following upper optimal upper bound
$$|D\mathcal{B}_p(x,t)| \leq ct^{-(n+1)/\theta}$$
- The intrinsic estimate above **exactly reproduces this upper bound**
- This decay estimate is indeed reproduced for all those solutions **that are initially compactly supported**

A priori estimates

- The previous bound always implies a priori estimates on standard parabolic cylinders

Theorem (Kuusi & Min.)

$$|Du(x,t)| \lesssim \left[\int_0^r \left(\frac{|\mu|(Q_\varrho(x,t))}{\varrho^{N-1}}\right)^{1/(p-1)} \frac{d\varrho}{\varrho}\right]^{p-1} + \fint_{Q_r(x,t)} (|Du|+s+1)^{p-1}\, dz$$

holds for every standard parabolic cylinder Q_r

A priori estimates

Theorem (Kuusi & Min.)

$$|Du(x,t)| \lesssim [\mathbf{W}_1^{\mu}(x,t;r)]^{p-1} + \fint_{Q_r(x,t)} (|Du| + s + 1)^{p-1}\, dx\, dt$$

holds for every standard parabolic cylinder Q_r

- The scaling deficit exponent $p-1$ appears

Occurrence of deficit scaling exponents

- For solutions to

$$u_t - \triangle_p u = \operatorname{div}\left(|F|^{p-2}F\right)$$

Theorem (Acerbi & Min., Duke Math. J. 2007)

$$\left(\fint_{Q_r} |Du|^q \, dz\right)^{\frac{1}{q}} \lesssim \left[\left(\fint_{Q_{2r}} |Du|^p \, dz\right)^{\frac{1}{p}} + \left(\fint_{Q_{2r}} |F|^q \, dz\right)^{\frac{1}{q}} + 1\right]^{\frac{p}{2}}$$

for every $q \geq p$

- The scaling deficit exponent $p/2$ appears

A priori estimates

Theorem (Kuusi & Min.)

It μ is time independent than

$$|Du(x,t)| \lesssim \mathbf{W}^{\mu}_{1/p,p}(x,r) + \fint_{Q_r} (|Du| + s + 1)^{p-1} \, dx \, dt$$

holds for every standard parabolic cylinder Q_r

A basic tool

Theorem (Kuusi & Min., General regularity estimate)

$$A, B, q \geq 1 \qquad \textit{and} \qquad \varepsilon \in (0,1)\,.$$

Then there exists a constant $\delta_\varepsilon \in (0, 1/2)$

$$\frac{\lambda}{B} \leq \sup_{Q^\lambda_{\delta_\varepsilon r}} \|Dw\| \leq \sup_{Q^\lambda_r} \|Dw\| \leq A\lambda$$

holds, then

$$E_q(Dw, \delta_\varepsilon Q^\lambda_r) \leq \varepsilon E_q(Dw, Q^\lambda_r)$$

holds, where

$$E_q(Dw, Q^\lambda_\varrho) := \left(\fint_{Q^\lambda_\varrho} |Dw - (Dw)_{Q^\lambda_\varrho}|^q \, dx\, dt \right)^{1/q}$$

Gradient continuity via potentials

Theorem (Kuusi & Min.)

Assume that

$$\lim_{r \to 0} \sup_{(x,t) \in \Omega_T} \mathbf{W}_1^{\mu}(x,t;r) = 0$$

holds, then Du is continuous in Q_T

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Theorem (Kuusi & Min.)

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Previous assumption reads as

$$\lim_{r\to 0} \sup_{(x,t)\in\Omega_T} \int_0^r \left(\frac{|\mu|(Q_\varrho(x,t))}{\varrho^{N-1}} \right)^{1/(p-1)} \frac{d\varrho}{\varrho} = 0$$

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$$\lim_{r\to 0} \sup_{(x,t)\in\Omega_T} \int_0^r \left(\frac{|\mu|(Q_\varrho(x,t))}{\varrho^{N-1}}\right)^{1/(p-1)} \frac{d\varrho}{\varrho} = 0$$

i.e. the convergence is uniform

A subtle fact

Theorem (Kuusi & Min., General regularity estimate)

$$A, B, q \geq 1 \qquad \textit{and} \qquad \varepsilon \in (0,1)\,.$$

Then there exists a constant $\delta_\varepsilon \in (0, 1/2)$

$$\frac{\lambda}{B} \leq \sup_{Q^\lambda_{\delta_\varepsilon r}} \|Dw\| \leq \sup_{Q^\lambda_r} \|Dw\| \leq A\lambda$$

holds, then

$$E_q(Dw, \delta_\varepsilon Q_r^\lambda) \leq \varepsilon E_q(Dw, Q_r^\lambda)$$

with

$$\delta_\gamma = \frac{1}{c(A)} \left(\frac{\varepsilon}{B}\right)^{1/\alpha} ,$$

Gradient continuity via potentials

Theorem (Kuusi & Min.)

Assume that

$$|\mu|(Q_\varrho) \lesssim \varrho^{N-1+\delta}$$

holds, then thtere exists α*, depending on* δ*, such that*

$$Du \in C^{0,\alpha} \qquad \textit{locally in} \ \ Q_T$$

Thanks for the attention (self-portrait of Serena Nono)