Blow up oscillating solutions to some nonlinear fourth order differential equations

Filippo GAZZOLA - Politecnico di Milano (Italy)

Joint work with Raffaella PAVANI

Madrid, 19 September 2011

GLOBAL SOLUTIONS

We consider the differential equation

$$w''''(s) + kw''(s) + f(w(s)) = 0 \qquad (s \in \mathbb{R})$$

where $k \in \mathbb{R}$ and

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad f(t)\, t > 0 \quad \text{for every } t \in \mathbb{R} \setminus \{0\}.$$

GLOBAL SOLUTIONS

We consider the differential equation

$$w''''(s) + kw''(s) + f(w(s)) = 0 \qquad (s \in \mathbb{R})$$

where $k \in \mathbb{R}$ and

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad f(t)\,t > 0 \quad \text{for every } t \in \mathbb{R} \setminus \{0\}.$$

THEOREM

(i) If a local solution w blows up at some finite $R \in \mathbb{R}$, then

$$\liminf_{s \to R} w(s) = -\infty \qquad \text{and} \qquad \limsup_{s \to R} w(s) = +\infty\,.$$

(ii) If f also satisfies

$$\limsup_{t \to +\infty} \frac{f(t)}{t} < +\infty \qquad \text{or} \qquad \limsup_{t \to -\infty} \frac{f(t)}{t} < +\infty,$$

then any local solution exists for all $s \in \mathbb{R}$.

- E. Berchio, A. Ferrero, F. Gazzola, P. Karageorgis, 2011

COMMENTS

COMMENTS

- If **both** the conditions

$$\limsup_{t\to+\infty} \frac{f(t)}{t} < +\infty \qquad \text{and} \qquad \limsup_{t\to-\infty} \frac{f(t)}{t} < +\infty,$$

are satisfied then global existence is straightforward.

COMMENTS

- If **both** the conditions

$$\limsup_{t\to+\infty} \frac{f(t)}{t} < +\infty \qquad \text{and} \qquad \limsup_{t\to-\infty} \frac{f(t)}{t} < +\infty,$$

are satisfied then global existence is straightforward.

- Under the sole assumption

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad f(t)\,t > 0 \quad \text{for every } t \in \mathbb{R} \setminus \{0\} \tag{1}$$

finite time blow up can occur only with **wide oscillations**.

COMMENTS

- If **both** the conditions

$$\limsup_{t\to+\infty} \frac{f(t)}{t} < +\infty \qquad \text{and} \qquad \limsup_{t\to-\infty} \frac{f(t)}{t} < +\infty,$$

are satisfied then global existence is straightforward.

- Under the sole assumption

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad f(t)\,t > 0 \quad \text{for every } t \in \mathbb{R} \setminus \{0\} \qquad (1)$$

finite time blow up can occur only with **wide oscillations**.

- The **first order** equation $w' + e^w - 1 = 0$ has the solution $w(s) = -\log(1 - e^{-s})$ which blows up as $s \searrow 0$.

COMMENTS

- If **both** the conditions

$$\limsup_{t\to+\infty}\frac{f(t)}{t}<+\infty \qquad \text{and} \qquad \limsup_{t\to-\infty}\frac{f(t)}{t}<+\infty,$$

are satisfied then global existence is straightforward.

- Under the sole assumption

$$f\in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R})\,,\quad f(t)\,t>0 \quad \text{for every } t\in\mathbb{R}\setminus\{0\} \qquad (1)$$

finite time blow up can occur only with **wide oscillations**.

- The **first order** equation $w'+e^w-1=0$ has the solution $w(s)=-\log(1-e^{-s})$ which blows up as $s\searrow 0$.

- The **first order** equation $w'+w+w^3=0$ has the solutions $w(s)=\frac{\gamma}{\sqrt{e^{2s}-\gamma^2}}$ $(\gamma\in\mathbb{R})$ which, if $\gamma\neq 0$, blow up as $s\searrow \log|\gamma|$. The blow up occurs monotonically.

• Consider the **second order** equation $w'' + f(w) = 0$ with f merely satisfying (1). Solution are concave (convex) whenever positive (negative). By studying the corresponding Hamiltonian system, any local solution is global.

• Consider the **second order** equation $w'' + f(w) = 0$ with f merely satisfying (1). Solution are concave (convex) whenever positive (negative). By studying the corresponding Hamiltonian system, any local solution is global.

• Consider instead the **second order** equation $-w'' + f(w) = 0$ with f satisfying (1); w is convex whenever it is positive and therefore it blows up monotonically in finite time if f is superlinear (Mitidieri-Pohožaev, 2001). In fact, the same occurs for the fourth order equation if we change sign to a superlinear f.

• Consider the **second order** equation $w'' + f(w) = 0$ with f merely satisfying (1). Solution are concave (convex) whenever positive (negative). By studying the corresponding Hamiltonian system, any local solution is global.

• Consider instead the **second order** equation $-w'' + f(w) = 0$ with f satisfying (1); w is convex whenever it is positive and therefore it blows up monotonically in finite time if f is superlinear (Mitidieri-Pohožaev, 2001). In fact, the same occurs for the fourth order equation if we change sign to a superlinear f.

• The **third order** equation $w''' + w^3 = 0$ admits the solutions $w(s) = \pm\sqrt{\frac{105}{8}}s^{-3/2}$ which are defined on $(0, +\infty)$ and blow up monotonically as $s \searrow 0$.

• Consider the **second order** equation $w'' + f(w) = 0$ with f merely satisfying (1). Solution are concave (convex) whenever positive (negative). By studying the corresponding Hamiltonian system, any local solution is global.

• Consider instead the **second order** equation $-w'' + f(w) = 0$ with f satisfying (1); w is convex whenever it is positive and therefore it blows up monotonically in finite time if f is superlinear (Mitidieri-Pohožaev, 2001). In fact, the same occurs for the fourth order equation if we change sign to a superlinear f.

• The **third order** equation $w''' + w^3 = 0$ admits the solutions $w(s) = \pm\sqrt{\frac{105}{8}}s^{-3/2}$ which are defined on $(0, +\infty)$ and blow up monotonically as $s \searrow 0$.

Finite time blow up forces wide oscillations

only in equations of order at least 4.

TWO QUESTIONS:

TWO QUESTIONS:

1) If $f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R})$ and $f(t)\,t > 0 \quad \forall t \in \mathbb{R} \setminus \{0\}$, then finite time blow up can occur only with wide oscillations. **Does it occur?**

TWO QUESTIONS:

1) If $f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R})$ and $f(t)\,t > 0 \quad \forall t \in \mathbb{R} \setminus \{0\}$, then finite time blow up can occur only with wide oscillations. **Does it occur?**

2) This is a PDE conference...

Nonlinear PDEs and Functional Inequalities Workshop
UAM Madrid (Spain), September 19-20, 2011

Are there some **applications to PDEs?**

TWO QUESTIONS:

1) If $f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R})$ and $f(t)\, t > 0 \quad \forall t \in \mathbb{R} \setminus \{0\}$, then finite time blow up can occur only with wide oscillations. **Does it occur?**

2) This is a PDE conference...

Nonlinear PDEs and Functional Inequalities Workshop
UAM Madrid (Spain), September 19-20, 2011

Are there some **applications to PDEs?**

TWO ANSWERS: YES & YES

THEOREM Assume that

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad \exists \lambda, \delta, \gamma > 0 \text{ s.t. } f(t)t \geq \delta t^2 + \lambda |t|^{2+\gamma} \quad \forall t \in \mathbb{R}.$$

Let w be a local solution to

$$w''''(s) + f(w(s)) = 0 \qquad (s \in \mathbb{R})$$

in a neighborhood of $s = 0$ such that

$$w'(0)w''(0) - w(0)w'''(0) > 0\,.$$

THEOREM Assume that

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad \exists \lambda, \delta, \gamma > 0 \text{ s.t. } f(t)t \geq \delta t^2 + \lambda |t|^{2+\gamma} \quad \forall t \in \mathbb{R}.$$

Let w be a local solution to

$$w''''(s) + f(w(s)) = 0 \qquad (s \in \mathbb{R})$$

in a neighborhood of $s = 0$ such that

$$w'(0)w''(0) - w(0)w'''(0) > 0 .$$

Let $R \in (0, +\infty]$ be the supremum of the maximal interval of continuation of w. Then $\exists \{s_j\}_{j \in \mathbb{N}}$ (increasing) such that:

THEOREM Assume that

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad \exists \lambda, \delta, \gamma > 0 \text{ s.t. } f(t)t \geq \delta t^2 + \lambda |t|^{2+\gamma} \quad \forall t \in \mathbb{R}.$$

Let w be a local solution to

$$w''''(s) + f(w(s)) = 0 \qquad (s \in \mathbb{R})$$

in a neighborhood of $s = 0$ such that

$$w'(0)w''(0) - w(0)w'''(0) > 0 \,.$$

Let $R \in (0, +\infty]$ be the supremum of the maximal interval of continuation of w. Then $\exists \{s_j\}_{j \in \mathbb{N}}$ (increasing) such that:
(i) $s_j \nearrow R$ as $j \to \infty$;

THEOREM Assume that

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad \exists \lambda, \delta, \gamma > 0 \text{ s.t. } f(t)t \geq \delta t^2 + \lambda |t|^{2+\gamma} \quad \forall t \in \mathbb{R}.$$

Let w be a local solution to

$$w''''(s) + f(w(s)) = 0 \qquad (s \in \mathbb{R})$$

in a neighborhood of $s = 0$ such that

$$w'(0)w''(0) - w(0)w'''(0) > 0\,.$$

Let $R \in (0, +\infty]$ be the supremum of the maximal interval of continuation of w. Then $\exists \{s_j\}_{j \in \mathbb{N}}$ (increasing) such that:
(i) $s_j \nearrow R$ as $j \to \infty$;
(ii) $\lim\limits_{j\to\infty} (s_{j+1} - s_j) = 0$;

THEOREM Assume that

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad \exists \lambda, \delta, \gamma > 0 \text{ s.t. } f(t)t \geq \delta t^2 + \lambda |t|^{2+\gamma} \quad \forall t \in \mathbb{R}.$$

Let w be a local solution to

$$w''''(s) + f(w(s)) = 0 \qquad (s \in \mathbb{R})$$

in a neighborhood of $s = 0$ such that

$$w'(0)w''(0) - w(0)w'''(0) > 0\,.$$

Let $R \in (0, +\infty]$ be the supremum of the maximal interval of continuation of w. Then $\exists \{s_j\}_{j\in\mathbb{N}}$ (increasing) such that:
(i) $s_j \nearrow R$ as $j \to \infty$;
(ii) $\lim_{j\to\infty}(s_{j+1} - s_j) = 0$;
(iii) $w(s_j) = 0$ and w has constant sign in (s_j, s_{j+1}) for all $j \in \mathbb{N}$;

THEOREM Assume that

$$f \in \mathrm{Lip}_{\mathrm{loc}}(\mathbb{R}), \quad \exists \lambda, \delta, \gamma > 0 \text{ s.t. } f(t)t \geq \delta t^2 + \lambda |t|^{2+\gamma} \quad \forall t \in \mathbb{R}.$$

Let w be a local solution to

$$w''''(s) + f(w(s)) = 0 \qquad (s \in \mathbb{R})$$

in a neighborhood of $s = 0$ such that

$$w'(0)w''(0) - w(0)w'''(0) > 0\,.$$

Let $R \in (0, +\infty]$ be the supremum of the maximal interval of continuation of w. Then $\exists \{s_j\}_{j\in\mathbb{N}}$ (increasing) such that:
(*i*) $s_j \nearrow R$ as $j \to \infty$;
(*ii*) $\lim_{j\to\infty} (s_{j+1} - s_j) = 0$;
(*iii*) $w(s_j) = 0$ and w has constant sign in (s_j, s_{j+1}) for all $j \in \mathbb{N}$;
(*iv*) $\max_{s\in[s_j,s_{j+1}]} |w(s)| \to +\infty$ as $j \to \infty$.

Assumptions:

f is superlinear $\quad + \quad k=0 \quad + \quad$ suitable initial condition.

Assumptions:

f is superlinear $\quad + \quad k = 0 \quad + \quad$ suitable initial condition.

Statement:

Any local solution has infinitely many oscillations which tend to enlarge width and to concentrate on small intervals.

Assumptions:
f is superlinear $\quad + \quad k = 0 \quad + \quad$ suitable initial condition.

Statement:
Any local solution has infinitely many oscillations which tend to enlarge width and to concentrate on small intervals.

This is not yet what we expected, but it gives a **strong hint**.

Why $k = 0$? Because the solution exhibits some nice qualitative behavior for $k \geq 0$ and some others for $k \leq 0$.

Why $k = 0$? Because the solution exhibits some nice qualitative behavior for $k \geq 0$ and some others for $k \leq 0$.

Moreover:

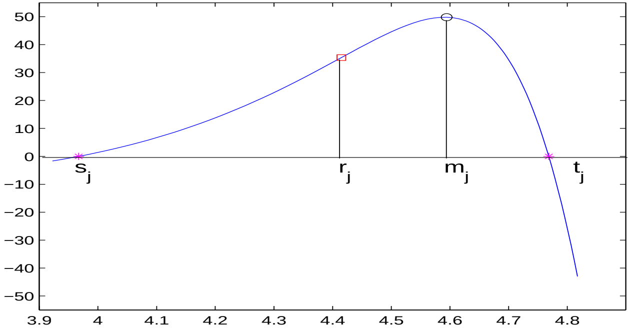


Figure: Qualitative behavior of the solution w in the interval $[s_j, t_j]$.

CRITICAL GROWTH BIHARMONIC EQUATIONS

For $n \geq 5$ consider the coercive elliptic equation

$$\Delta^2 u + |u|^{8/(n-4)} u = 0 \qquad \text{in } \mathbb{R}^n.$$

CRITICAL GROWTH BIHARMONIC EQUATIONS

For $n \geq 5$ consider the coercive elliptic equation

$$\Delta^2 u + |u|^{8/(n-4)} u = 0 \qquad \text{in } \mathbb{R}^n.$$

The corresponding noncoercive equation $\Delta^2 u - |u|^{8/(n-4)} u = 0$ has been extensively studied (Swanson 1992, Gazzola-Grunau 2006, Lazzo-Schmidt 2009, ...).

CRITICAL GROWTH BIHARMONIC EQUATIONS

For $n \geq 5$ consider the coercive elliptic equation

$$\Delta^2 u + |u|^{8/(n-4)} u = 0 \qquad \text{in } \mathbb{R}^n.$$

The corresponding noncoercive equation $\Delta^2 u - |u|^{8/(n-4)} u = 0$ has been extensively studied (Swanson 1992, Gazzola-Grunau 2006, Lazzo-Schmidt 2009, ...).

THEOREM Let $n \geq 5$ and let $u = u(r)$ be a nontrivial radially symmetric solution to the above equation in a neighborhood of the origin. Then there exists $\rho \in (0, \infty)$ such that

$$\liminf_{r \nearrow \rho} u(r) = -\infty \qquad \text{and} \qquad \limsup_{r \nearrow \rho} u(r) = +\infty \, .$$

TOOLS IN THE PROOF

TOOLS IN THE PROOF

- Transformation of the equation by Gazzola-Grunau (2006)

TOOLS IN THE PROOF

- Transformation of the equation by Gazzola-Grunau (2006)
- Some computations by Lazzo-Schmidt (2009)

TOOLS IN THE PROOF

• Transformation of the equation by Gazzola-Grunau (2006)

• Some computations by Lazzo-Schmidt (2009)

• The before mentioned result by Berchio-Ferrero-Gazzola-Karageorgis (2011)

TOOLS IN THE PROOF

- Transformation of the equation by Gazzola-Grunau (2006)
- Some computations by Lazzo-Schmidt (2009)
- The before mentioned result by Berchio-Ferrero-Gazzola-Karageorgis (2011)
- A Liouville-type result by D'Ambrosio-Mitidieri (2012?)

THEOREM Let $n \geq 5$ be an integer. There exists a solution $w = w(s)$ to the equation

$$w''''(s) - \frac{n^2-4n+8}{2} w''(s) + \left(\frac{n(n-4)}{4}\right)^2 w(s) + |w(s)|^{8/(n-4)} w(s) = 0$$

which is defined in a neighborhood of $s = -\infty$ and such that

$$\liminf_{s\to R} w(s) = -\infty \qquad \text{and} \qquad \limsup_{s\to R} w(s) = +\infty$$

for some finite $R \in \mathbb{R}$.

THEOREM Let $n \geq 5$ be an integer. There exists a solution $w = w(s)$ to the equation

$$w''''(s) - \frac{n^2-4n+8}{2} w''(s) + \left(\frac{n(n-4)}{4}\right)^2 w(s) + |w(s)|^{8/(n-4)} w(s) = 0$$

which is defined in a neighborhood of $s = -\infty$ and such that

$$\liminf_{s \to R} w(s) = -\infty \qquad \text{and} \qquad \limsup_{s \to R} w(s) = +\infty$$

for some finite $R \in \mathbb{R}$.

The nonlinearity $f(t) = \left(\frac{n(n-4)}{4}\right)^2 t + |t|^{8/(n-4)} t$ satisfies the sign condition (1).

When $n = 8$, the previous equation becomes
$w''''(s) - 20w''(s) + 64w(s) + w(s)^3 = 0.$

When $n = 8$, the previous equation becomes
$w''''(s) - 20w''(s) + 64w(s) + w(s)^3 = 0$.

NUMERICAL RESULTS The first 18 zeros of the solution w satisfying $[w(0), w'(0), w''(0), w'''(0)] = [1, 0, 0, 0]$ are given by:

$z_1 = 0.716$, $z_2 = 1.7977$, $z_3 = 2.13827$, $z_4 = 2.17358$, $z_5 = 2.18718$,
$z_6 = 2.192412$, $z_7 = 2.194429$, $z_8 = 2.1952053$, $z_9 = 2.1955044$,
$z_{10} = 2.1956196$, $z_{11} = 2.19566400$, $z_{12} = 2.19568109$,
$z_{13} = 2.195687680$, $z_{14} = 2.195690216$, $z_{15} = 2.1956911931$,
$z_{16} = 2.1956915694$, $z_{17} = 2.19569171433$, $z_{18} = 2.19569177015$.

When $n = 8$, the previous equation becomes
$w''''(s) - 20w''(s) + 64w(s) + w(s)^3 = 0$.

NUMERICAL RESULTS The first 18 zeros of the solution w satisfying $[w(0), w'(0), w''(0), w'''(0)] = [1, 0, 0, 0]$ are given by:

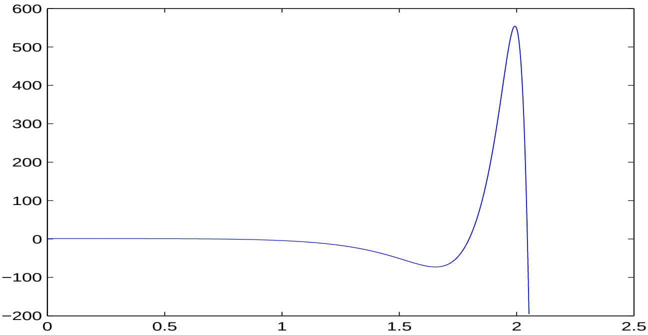
$z_1 = 0.716$, $z_2 = 1.7977$, $z_3 = 2.13827$, $z_4 = 2.17358$, $z_5 = 2.18718$,
$z_6 = 2.192412$, $z_7 = 2.194429$, $z_8 = 2.1952053$, $z_9 = 2.1955044$,
$z_{10} = 2.1956196$, $z_{11} = 2.19566400$, $z_{12} = 2.19568109$,
$z_{13} = 2.195687680$, $z_{14} = 2.195690216$, $z_{15} = 2.1956911931$,
$z_{16} = 2.1956915694$, $z_{17} = 2.19569171433$, $z_{18} = 2.19569177015$.

Moreover the first 16 critical levels are given by

1.00000e+000	-7.28173e+001	**5.54303e+002**	-3.79831e+003
2.56635e+004	-1.73041e+005	**1.16639e+006**	-7.86188e+006
5.29910e+007	-3.57173e+008	**2.40743e+009**	-1.62267e+010
1.09371e+011	-7.37197e+011	**4.96887e+012**	-3.34914e+013

The blow up time seems to be $\overline{s} = 2.1957$ (rounded to 5 significant digits).

We plot the computed solution until $s = 2.05281$, i.e. just a little after the second relative maximum point.



SUSPENSION BRIDGES

SUSPENSION BRIDGES

The following nonlinear beam equation was proposed by Lazer-McKenna (1990) as a model for a suspension bridge

$$u_{tt} + u_{xxxx} + \gamma u^+ = W(x,t) \qquad x \in (0,L)\,, \quad t > 0\,,$$

• $L > 0$ denotes the length of the bridge;
• γu^+ represents the force due to the cables which are considered as a spring with a one-sided restoring force (equal to γu if u is downward positive and to 0 if u is upward negative);
• W represents the forcing term acting on the bridge (including its own weight per unit length and the wind or other external sources);
• the solution u represents the vertical displacement when the beam is bending.

Normalizing the PDE by putting $\gamma = 1$ and $W \equiv 1$, and seeking traveling waves $u(x,t) = 1 + w(x - ct)$ leads to the ODE

$$w''''(s) + kw''(s) + [w(s) + 1]^+ - 1 = 0 \qquad (s \in \mathbb{R},\ k = c^2)\,.$$

Normalizing the PDE by putting $\gamma = 1$ and $W \equiv 1$, and seeking traveling waves $u(x,t) = 1 + w(x - ct)$ leads to the ODE

$$w''''(s) + kw''(s) + [w(s)+1]^+ - 1 = 0 \qquad (s \in \mathbb{R},\ k = c^2)\,.$$

In order to maintain the same behavior but with a smooth nonlinearity, Chen-McKenna (1997) suggest to consider the equation

$$w''''(s) + kw''(s) + e^{w(s)} - 1 = 0 \qquad (s \in \mathbb{R})\,,$$

which is exactly of our kind with $f(t) = e^t - 1$:
sign condition OK
one sided sublinear OK.

Normalizing the PDE by putting $\gamma = 1$ and $W \equiv 1$, and seeking traveling waves $u(x,t) = 1 + w(x - ct)$ leads to the ODE

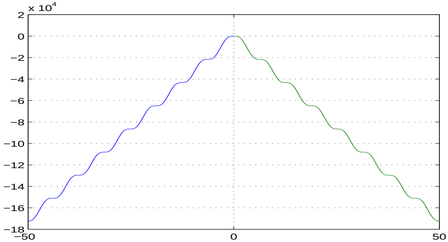
$$w''''(s) + kw''(s) + [w(s) + 1]^+ - 1 = 0 \qquad (s \in \mathbb{R},\ k = c^2)\,.$$

In order to maintain the same behavior but with a smooth nonlinearity, Chen-McKenna (1997) suggest to consider the equation

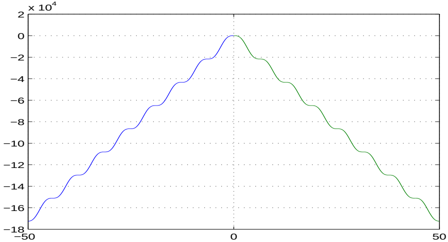
$$w''''(s) + kw''(s) + e^{w(s)} - 1 = 0 \qquad (s \in \mathbb{R})\,,$$

which is exactly of our kind with $f(t) = e^t - 1$:
sign condition OK
one sided sublinear OK.

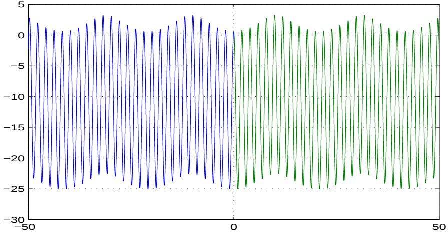
Hence, we know that any local solution is global.



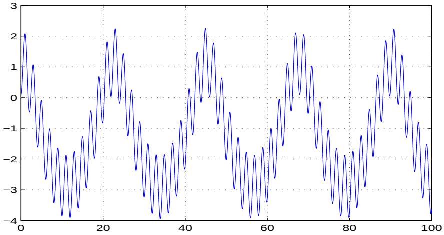
$$k = 1,\ [w(0), w'(0), w''(0), w'''(0)] = [10, 0, -10, 0].$$



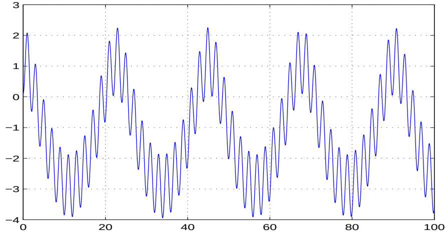
$$k = 1,\ [w(0), w'(0), w''(0), w'''(0)] = [10, 0, -10, 0].$$



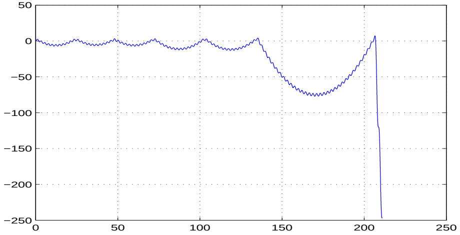
$$k = 10,\ [w(0), w'(0), w''(0), w'''(0)] = [0.6, 0, -128, 0].$$



$$k = 10,\ [w(0), w'(0), w''(0), w'''(0)] = [0.1, 0, 10, 0].$$



$$k = 10,\ [w(0), w'(0), w''(0), w'''(0)] = [0.1, 0, 10, 0].$$



$$k = 8,\ [w(0), w'(0), w''(0), w'''(0)] = [0.1, 0, 10, 0].$$

We wish to suggest **a variant of the suspension bridge model**.

We wish to suggest **a variant of the suspension bridge model**.

We wish to suggest **a variant of the suspension bridge model**.

The ratio between the length and the width of the Golden Gate Bridge suggests that suspension bridges may be modeled as a one dimensional beam. But...

We wish to suggest **a variant of the suspension bridge model**.

The ratio between the length and the width of the Golden Gate Bridge suggests that suspension bridges may be modeled as a one dimensional beam. But...

...the Tacoma Bridge collapse was due to a wide torsional motion: **the bridge cannot be considered as a one dimensional beam.**

As pointed out by McKenna (2006), according to historical sources, one of the most interesting behaviors for suspension bridges (including the Golden Gate and the Tacoma Narrows Bridge) is the following:

large vertical oscillations can rapidly change,
almost instantaneously, to a torsional oscillation.

As pointed out by McKenna (2006), according to historical sources, one of the most interesting behaviors for suspension bridges (including the Golden Gate and the Tacoma Narrows Bridge) is the following:

large vertical oscillations can rapidly change,
almost instantaneously, to a torsional oscillation.

Our explanation to this fact is:

since the motion cannot be continued downwards due to the cables, when the cables reach their maximum extension the existing energy generates a crossing wave, namely a torsional oscillation.

As pointed out by McKenna (2006), according to historical sources, one of the most interesting behaviors for suspension bridges (including the Golden Gate and the Tacoma Narrows Bridge) is the following:

large vertical oscillations can rapidly change,
almost instantaneously, to a torsional oscillation.

Our explanation to this fact is:

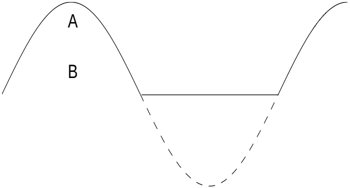
since the motion cannot be continued downwards due to the cables, when the cables reach their maximum extension the existing energy generates a crossing wave, namely a torsional oscillation.

For a different model, Drábek-Holubová-Matas-Necesal (2003) introduce the **deflection from horizontal** as a second unknown function.

We **maintain the one dimensional model provided one also allows displacements below the equilibrium position**. Then

We **maintain the one dimensional model provided one also allows displacements below the equilibrium position**. Then
$w =$ vertical displacement $+$ deflection from horizontal
and f is unbounded also from below.

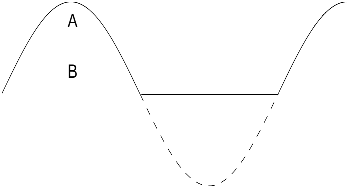
We **maintain the one dimensional model provided one also allows displacements below the equilibrium position**. Then
$w =$ vertical displacement $+$ deflection from horizontal
and f is unbounded also from below.



Dotted line = theoretical position of the bridge in absence of the action of the cables.

Horizontal line = position of the bridge when stopped by the cables.

We **maintain the one dimensional model provided one also allows displacements below the equilibrium position**. Then
$w =$ vertical displacement + deflection from horizontal
and f is unbounded also from below.

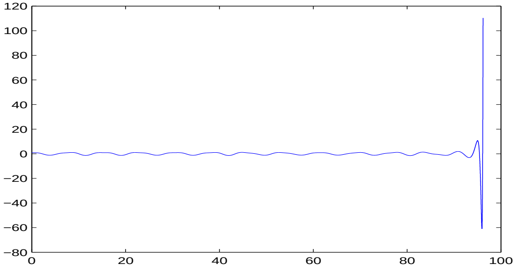


Dotted line = theoretical position of the bridge in absence of the action of the cables.

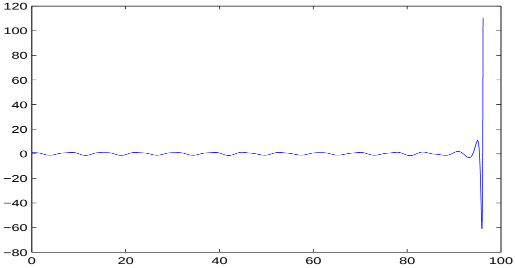
Horizontal line = position of the bridge when stopped by the cables.

More the position of the bridge is far from the horizontal equilibrium position, more the action of the wind becomes relevant because the wind hits transversally the surface of the bridge. In the limit vertical position, the wind would hit it orthogonally. Hence, f is superlinear, becoming more powerful for large displacements from the horizontal position: in position A the impact of the wind is much more relevant than in position B.

The solution to $w''''(s) + 3.6\, w''(s) + w(s) + w(s)^3 = 0$ with $[w(0), w'(0), w''(0), w'''(0)] = [0.9, 0, 0, 0]$



The solution to $w''''(s) + 3.6\, w''(s) + w(s) + w(s)^3 = 0$ with $[w(0), w'(0), w''(0), w'''(0)] = [0.9, 0, 0, 0]$



For the equation $w''''(s) + kw''(s) + w(s) + w(s)^3 = 0$ with $[w(0), w'(0), w''(0), w'''(0)] = [\alpha, 0, 0, 0]$ our results suggest that the blow up time $\overline{s}$ is decreasing w.r.t. $\alpha > 0$ (as expected) and increasing w.r.t. $k \in \mathbb{R}$. For $k \gg 1$ and/or for $\alpha \sim 0^+$ our numerical procedure shows a somehow periodic behavior: we do not know if the solution is indeed periodic or if the blow up time is so large that the numerical procedure does not reach it with sufficient precision.

Asymmetric nonlinearity. The solution to

$$w''''(s) + 2w''(s) + \frac{w(s) + w(s)^3 + e^{w(s)} - 1}{2} = 0 \qquad (s \in \mathbb{R})$$

with initial conditions $[w(0), w'(0), w''(0), w'''(0)] = [1, 0, 0, 0]$ exhibits blow up at $\overline{s} = 6.3009^+$.

Asymmetric nonlinearity. The solution to

$$w''''(s) + 2w''(s) + \frac{w(s) + w(s)^3 + e^{w(s)} - 1}{2} = 0 \qquad (s \in \mathbb{R})$$

with initial conditions $[w(0), w'(0), w''(0), w'''(0)] = [1, 0, 0, 0]$ exhibits blow up at $\overline{s} = 6.3009^+$. The relative extrema $s < \overline{s}$ estimated numerically are reported in

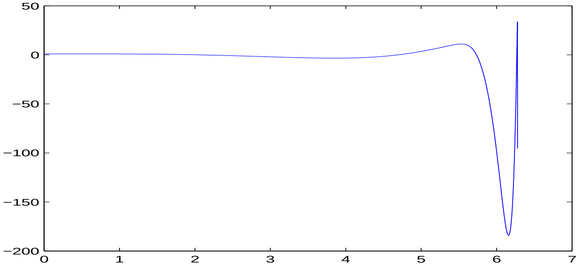
s	**0.0**	3.85325	**5.53421**	6.15714	**6.27537**	6.29695	**6.30086**	6.30091
$w(s)$	**1.0**	-3.3786	**11.055**	-184.06	**33.554**	-1.5026e+005	**73.377**	-2.3179e+010

Asymmetric nonlinearity. The solution to

$$w''''(s) + 2w''(s) + \frac{w(s) + w(s)^3 + e^{w(s)} - 1}{2} = 0 \qquad (s \in \mathbb{R})$$

with initial conditions $[w(0), w'(0), w''(0), w'''(0)] = [1, 0, 0, 0]$ exhibits blow up at $\overline{s} = 6.3009^+$. The relative extrema $s < \overline{s}$ estimated numerically are reported in

s	**0.0**	3.85325	**5.53421**	6.15714	**6.27537**	6.29695	**6.30086**	6.30091
$w(s)$	**1.0**	-3.3786	**11.055**	-184.06	**33.554**	-1.5026e+005	**73.377**	-2.3179e+010



As our Theorems and Numerical Results suggest, fourth order equations with "two-sided superlinear" nonlinearities exhibit traveling waves which blow up in finite time after a long waiting time of apparent calm and sudden wide oscillations.

As our Theorems and Numerical Results suggest, fourth order equations with “two-sided superlinear” nonlinearities exhibit traveling waves which blow up in finite time after a long waiting time of apparent calm and sudden wide oscillations.

Is this the explanation of the Tacoma Bridge collapse?

As our Theorems and Numerical Results suggest, fourth order equations with "two-sided superlinear" nonlinearities exhibit traveling waves which blow up in finite time after a long waiting time of apparent calm and sudden wide oscillations.

Is this the explanation of the Tacoma Bridge collapse?

THANK YOU FOR YOUR ANSWERS AND COMMENTS!

As our Theorems and Numerical Results suggest, fourth order equations with “two-sided superlinear” nonlinearities exhibit traveling waves which blow up in finite time after a long waiting time of apparent calm and sudden wide oscillations.

Is this the explanation of the Tacoma Bridge collapse?

THANK YOU FOR YOUR ANSWERS AND COMMENTS!

THANK YOU FOR YOUR ATTENTION!