

Courses 2016-17

BCAM Mazarredo 14 ,48009 Bilbao, Basque Country, Spain

June 26-30, 2017, (10:00 - 12:00)

(5 sessions, a total of 10 hours)

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THE FRACTIONAL LAPLACIAN: PROBABILISTIC STRUCTURE AND EXTENSIONS RELATED TO LÉVY PROCESSES

We will discuss selected topics related to the fractional Laplacian $\Delta^{\alpha/2}$. We will construct from the first principles the semigroup of the operator and the corresponding stochastic process—the isotropic α -stable Lévy process in \mathbb{R}^d [5]. Here $0 < \alpha < 2$ and $d = 1, 2, \dots$. The underlying probabilistic structure facilitates definitions and leads to quick applications in analysis. It also generalizes easily to other nonlocal operators/Lévy processes. We will define the Green function and Poisson kernel of $\Delta^{\alpha/2}$ for open sets $D \subset \mathbb{R}^d$ and explain their interconnections [3, 4, 10, 9]. I plan to cover some specific applications with focus on the Lévy systems [10]. If time permits, I will present applications to Fourier multipliers on L^p , which can be obtained via martingale transforms of Lévy processes [1]. Other developments may also be mentioned [8, 7, 6, 2] depending on the interests of the audience.

Prerequisites: Functional analysis, Partial Differential Equations and Probability, basic knowledge.

References:

- [1] R. Bañuelos and K. Bogdan. Lévy processes and Fourier multipliers. *J. Funct. Anal.*, 250(1):197–213, 2007.
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- [4] K. Bogdan and T. Byczkowski. Potential theory of Schrödinger operator based on fractional Laplacian. *Probab. Math. Statist.*, 20(2, Acta Univ. Wratislav. No. 2256):293–335, 2000.
- [5] K. Bogdan, T. Byczkowski, T. Kulczycki, M. Ryznar, R. Song, and Z. Vondraček. Potential analysis of stable processes and its extensions, volume 1980 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 2009. Edited by Piotr Graczyk and Andrzej Stós.
- [6] K. Bogdan and B. Dyda. The best constant in a fractional Hardy inequality. *Math. Nachr.*, 284(5-6):629–638, 2011.
- [7] K. Bogdan, T. Grzywny, and M. Ryznar. Heat kernel estimates for the fractional Laplacian with Dirichlet conditions. *Ann. Probab.*, 38(5):1901–1923, 2010.
- [8] K. Bogdan and T. Jakubowski. Estimates of heat kernel of fractional Laplacian perturbed by gradient operators. *Comm. Math. Phys.*, 271(1):179–198, 2007.
- [9] K. Bogdan, T. Kulczycki, and M. Kwasnicki. Estimates and structure of α -harmonic functions. *Probab. Theory Related Fields*, 140(3-4):345–381, 2008.
- [10] K. Bogdan, J. Rosinski, G. Serafin, and Ł. Wojciechowski. Lévy systems and moment formulas for mixed Poisson integrals. *ArXiv e-prints*, Nov. 2014.

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