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Theta functions on the Boundary of Moduli Space

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ABSTRACT. By approaching singular stable curves (Riemann surfaces with nodes) through nonsingular ones, we work out some aspects of the theory of abelian integrals and theta functions.

The topics dealt with are period matrices, embedding a curve into its jacobian, Riemann's theta function Lefschetz theorem and solutions of P.D.E's of K-dV type.

We work within the framework of Bers' theory of deformation spaces.

0. Introduction

Let $D = D(S)$ be Bers' deformation space of a Riemann surface with nodes (stable curve) S. If S has no nodes at all D is simply T_g, the Teichmüller space; in this case it is well known that there are fibre spaces $\pi\colon V_g \to T_g$ such that $\pi^{-1}(t) = S_t$ is the Riemann surface represented by $t \in T_g$, and $\pi_1\colon J(V_g) \to T_g$ such that $\pi_1^{-1}(t)$ is the jacobian of S_t, i.e. $\pi_1^{-1}(t) = J(S_t) = \mathbf{C}^g/\mathbf{Z}^g + \mathbf{Z}^g \cdot \Omega_t$ with Ω the period matrix.

One each $J(S_t)$ $t \in T_g$; one has the classical theta function $\theta(z, \Omega_t)$. This article is an attempt to understand what happens to this function when we allow t to vary in D (not only in T_g).

We start by looking at a holomorphic map of fibre spaces over T_g defined by Earle in [E],

$$\Phi\colon \begin{array}{ccc} V_g & \to & J(V_g) \\ & {}_{\pi}\searrow \quad \swarrow_{\pi_1} & \\ & T_g & \end{array}; \qquad \Phi(t, x) = \left(t, \frac{1}{1-g} k(t, x)\right),$$

$$k(t, x) = \text{Riemann's constant},$$

which embeds each nonsingular curve into its jacobian. We observe that in order to extend this map to singular stable curves one has to translate the

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