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ON A BASIS FOR $H_2(\overline{M}_g)$

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ABSTRACT. In the moduli space $\overline{\mathbf{M}}_g$ of stable curves (Riemann surfaces with nodes) we construct a basis for the second homology group, which is dual to the standard basis for the second cohomology group. The elements of our basis are algebraic curves.

0. INTRODUCTION

Let $\mathbf{M}_g$ denote the moduli space of nonsingular complex curves (compact Riemann surfaces) of genus g and $\overline{\mathbf{M}}_g$ its stable curves compactification. The goal of this note is to provide a set of $2+[g/2]$ complete curves (compact analytic subspaces of complex dimension 1), $E, E_0, \ldots, E_{[g/2]}$ that pair diagonally with the basic divisor classes $\lambda, \delta_0, \ldots, \delta_{[g/2]}$ (see [HM]). A fundamental result of Harer (see [Wo]) implies at once that these curves afford a basis for $H_2(\overline{\mathbf{M}}_g)$.

From now on, we restrict ourselves to $g > 3$, because our method to construct a curve E_0 dual to D_0 does not work for $g = 3$ (but see Remark 1). As for the curves E_i $i \geq 1$, they are, of course, those introduced by Mumford and Harris [HM].

Our work is closely related to the article of Wolpert [Wo]. As in that paper we employ the Teichmüller coordinates for $\overline{\mathbf{M}}_g$ defined by Bers in [Be]. Apart from the fact that our intersection table is diagonal, our method is different in that we avoid homotopy theory. Instead we determine the restriction of these divisors to our chosen curves. In this way, we remain within the framework of analytic (algebraic) geometry.

1. PRELIMINARIES

1.1. $\overline{\mathbf{M}}_g$ is the moduli space of stable curves (Riemann surfaces with nodes [Be]). It is a compactification of $\mathbf{M}_g$ whose compactification locus, $\overline{\mathbf{M}}_g - \mathbf{M}_g$, is the union of $1+[g/2]$ divisors $D_0, \ldots, D_{[g/2]}$. These can be readily described

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