LOCI OF CURVES WHICH ARE PRIME GALOIS COVERINGS OF $\mathbb{P}^1$

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[Received 4 August 1989—Revised 17 May 1990]

0. *Introduction*

Torelli's problem for curves (compact Riemann surfaces), in its sharpest form, could be formulated as follows:

> 'Given the period matrix of a Riemann surface S, write down an algebraic equation $f(x, y) = 0$ whose associated Riemann surface is precisely (isomorphic to) S.'

For hyperelliptic curves $y^2 = (x - a_1) \dots (x - a_r)$, it is well known that one can express the a_i in terms of functions (theta functions) of its period matrix (see, for example, [**18**, **6**]).

One of our goals here is to exhibit similar expressions for curves of the form

$$y^p = (x - a_1)^{m_1} \dots (x - a_r)^{m_r}, \quad \text{with } p \text{ prime.} \tag{0.1}$$

The other aim is to study the moduli of these curves.

Our work is suggested by Mumford's analysis of the case where $p = 2$ carried out in [**18**, § 8]. The approach is different in that we make extensive use of Teichmüller theory. For given p, r and integers m_i, there is a Teichmüller space Λ parametrising (marked) curves of the form (0.1). On it, we shall construct functions

$$\lambda_i\colon \Lambda \to \mathbb{C},$$
$$t \to \lambda_i(t),$$

where the $\lambda_i(t)$ are the quotients of theta constants for $i = 3, 4, \dots, r$, such that

$$y^p = x^{m_1}(x-1)^{m_2} \dots (x - \lambda_r(t))^{m_r}$$

is the Riemann surface represented by $t \in \Lambda$. Thus, the λ_i generalise in a natural way the classical lambda function. Then, by studying the behaviour of the λ_i under modular action, we are able to describe our moduli spaces. In this way we recover classical ($p = 2$) as well as recent ($p = 3$ [**2**]) results.

This study also allows us to deal with questions such as normality and rationality.

The reader is assumed to be familiar with the theory (*and notation*) of Riemann surfaces and theta functions for which we refer to [**17**] and [**6**].

I would like to express my gratitude to W. J. Harvey who, among other things, suggested the proof of the crucial Lemma 5.2 to me.

Research supported by a grant of the CICYT, M.E.C., Spain.

A.M.S. (1980) subject classification: 14H15, 32G15, 14K25.

Proc. London Math. Soc (3) 62 (1991) 469–489.