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On complete curves in moduli space II

BY GABINO GONZÁLEZ DÍEZ
Departamento de Matemáticas, Universidad Autónoma de Madrid, Spain

AND WILLIAM J. HARVEY
Department of Mathematics, King's College London

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Introduction

In a companion to this article [**4**], we gave a construction which furnishes complete curves, inside the moduli variety $\mathscr{M}_g$ of non-singular genus g curves, for each $g \geqslant 4$; it was also shown there that the technique will not work in genus 3.

The purpose of this article is to furnish a complete curve in $\mathscr{M}_3$ using a totally different method based on the theory of theta functions and the vanishing properties of automorphic forms on Siegel space defined by theta constants.

Certain preliminary results valid in arbitrary genus are proved, but these techniques do not all appear to extend to higher genus and it remains an open problem to produce explicit constructions of the multitude of complete curves known to exist through every finite set of points (see [**5**] for further discussion of this point).

1. *Review of Satake's compactification*

Let us denote by $\mathbf{H}_g$ the set of $g \times g$ complex matrices Ω which are symmetric and have positive definite imaginary part. The *symplectic group* $\mathrm{Sp}(g;\mathbb{Z})$ acts properly discontinuously on $\mathbf{H}_g$ by

$$M.\Omega = (A\Omega+B)(C\Omega+D)^{-1}$$

where

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \mathrm{Sp}(g,\mathbb{Z}).$$

The quotient space $\mathbf{A}_g = \mathbf{H}_g/\mathrm{Sp}(g,\mathbb{Z})$ is the *Siegel space*.

Recall that for $g \geqslant 2$, a *Siegel modular form* of degree k is a holomorphic function on $\mathbf{H}_g$ satisfying

$$F(M.\Omega) = \det(C\Omega+D)^k F(\Omega).$$

The fundamental result we require is that by taking a basis $F_0, \ldots, F_n$ of the vector space of modular forms of suitable (fixed) weight k, the map $\Omega \mapsto (F_0(\Omega), \ldots, F_n(\Omega))$ embeds Siegel space in projective space (see [**1**, **3**, **9**]).

Let us denote by $\hat{\mathbf{A}}_g$ the Zariski closure of $\mathbf{A}_g$ via this embedding. $\hat{\mathbf{A}}_g$ is called the *Satake compactification* of Siegel space in honour of Satake, who described it as the disjoint union

$$\hat{\mathbf{A}}_g = \mathbf{A}_g \cup \mathbf{A}_{g-1} \cup \ldots \cup \mathbf{A}_1 \cup \mathbf{A}_0 = \mathbf{A}_g \cup \hat{\mathbf{A}}_{g-1}.$$

We do not need to give a detailed account of Satake's topology; it will be sufficient