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ON THE NUMBER OF COINCIDENCES OF MORPHISMS BETWEEN CLOSED RIEMANN SURFACES

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Abstract

We give a bound for the number of coincidences of two morphisms between given compact Riemann surfaces (complete complex algebraic curves). Our results generalize well known facts about the number of fixed points of an automorphism.

Let M be a compact Riemann surface (complete complex algebraic curve) of genus $g \geq 2$, and $\tau : M \to M$ an automorphism different from the identity. Then it is well known (see e.g. [**F-K**]) that τ has at most $2g+2$ fixed points and that this bound is attained if and only if M is hyperelliptic and τ is the hyperelliptic involution.

With this in mind, we consider two distinct morphisms $f_i : M \to M'$ of degrees d_i $(i = 1, 2)$ between compact Riemann surfaces of genera g and $g' \geq 2$ respectively, and look at the number of *coincidences*, that is, the number of points at which f_1 and f_2 agree.

The result we obtain (Theorem 2.9) is that f_1 and f_2 have at most $d_1 + 2g'\sqrt{d_1 d_2} + d_2$ coincidences, and that this number (suitably counted) is attained if and only if M' is hyperelliptic and f_1 and f_2 differ by composition with the hyperelliptic involution. When these morphisms are isomorphisms, i.e. when $d_1 = d_2 = 1$, then, of course, the coincidences are the fixed points of the automorphism $\tau = f_1^{-1} \circ f_2$; in this case our result agrees with the classical one.

The proof uses a Lefschetz trace formula for the case of two morphisms, which is a straightforward generalization of the standard one and, no doubt, is well known to topologists. However, at least in the precise form we need it here, we have not been able to locate it in the literature

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