

On Prime Galois Coverings of the Riemann Sphere (*) (**).

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0. – Introduction and statement of results.

1. Let C be a complex curve (compact Riemann surface) of genus $g \geq 2$. We shall be primarily dealing with holomorphic automorphisms τ of prime order p such that the quotient surface $C/\langle\tau\rangle$ is isomorphic to P^1 .

If $p = 2$ the curve is said to be hyperelliptic and τ , which is then called the hyperelliptic involution, is known to be the unique automorphism of order 2 such that $C/\langle\tau\rangle \cong P^1$. The situation is quite different when $p > 2$, for then well-known curves such as Klein's and Fermat's provide examples of curves admitting different automorphisms of same prime order with quotient P^1 (Examples 1.1, 1.2, 1.3). However, in all these examples we easily see that groups generated by such automorphisms are conjugate. The first part of this article is devoted to prove that this holds always; namely we shall prove.

THEOREM 1. – If the automorphism group of a Riemann surface S $\text{Aut}(S)$, contains automorphisms τ_1, τ_2 of same prime order and such that the quotient surfaces $S/\langle\tau_i\rangle$ ($i = 1, 2$) are isomorphic to P^1 ; then $\langle\tau_1\rangle$ and $\langle\tau_2\rangle$ are conjugate in $\text{Aut}(S)$.

This result can be viewed as a generalisation of the uniqueness of the hyperelliptic involution. We shall also provide examples to show that our assumptions on the order and the quotient are necessary (Examples 1.9, 1.10).

2. In the rest of the paper we shall investigate the implications of Theorem 1 in the geometry of moduli (and Teichmüller) space. In order to describe our results we need to introduce some notation.

Let $P^n(F_p)$ denote the projective space over the field with p elements F_p , and define the following subset D_p^r of $P^{r-1}(F_p)$,

$$D_p^r = \left\{ m = (m_1, \dots, m_r) \mid \sum_{i=1}^r m_i = 0 \text{ and } \prod_{i=1}^r m_i \neq 0 \right\}.$$

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