CORRIGENDUM TO "FIELDS OF MODULI AND DEFINITION OF HYPERELLIPTIC COVERS"

BY

YOLANDA FUERTES AND GABINO GONZÁLEZ-DIEZ

Consider the algebraic curve

$$C:\ y^2=\prod_{d=4}^{2g+2}\left(x^4-2\left(1-2\frac{r_3-r_1}{r_3-r_2}\frac{q_d-r_2}{q_d-r_1}\right)x^2+1\right)$$

where r_1, r_2, r_3 are the roots of the polynomial x^3-3x+1 (or any other degree 3 polynomial $p(x)\in\mathbb{Q}[x]$ whose Galois group has order 3) and the parameters q_i are distinct rational numbers $q_4,...,q_{2g+2}$ chosen so that $Aut(C)\cong\mathbb{Z}_2\oplus\mathbb{Z}_2\oplus\mathbb{Z}_2$. Then

Theorem 1. *I) C is hyperelliptically defined over $\mathbb{Q}(r_1)$.*

II) The field of moduli of C is $\mathbb{Q}$.

III) Let k be a subfield of the reals and C_k a curve of the form C_k: $y^2=q(x)$, where $q(x)$ is a polynomial with coefficients in k without multiple roots. Suposse that there is a birational isomorphism $f: C\to C_k$ defined over the the compositum of the fields $\mathbb{Q}(r_1)$ and k, namely $k(r_1)$. Then k must contain the field $\mathbb{Q}(r_1)$.

This is a correction to Theorem 14 in [1] in which the stament III) was claimed to hold without imposing any restriction to the field of definition of f. In the proof we took a point $(a,b)\in C$ such that $f(a,b)=(x,y)$ with $x\in\mathbb{Q}$ (or, for that matter, in $k(r_1)$) and claimed that the field $k(r_1,i,a)$ is a Galois extension of k. This is not at all clear. However if f is assumed to be defined over $k(r_1)$ then the point $(a,b)=(0,1)$ clearly satisfies the condition since in that case $k(r_1,i,a)=k(r_1,i)$ and only the first of the cases discussed in Proposition 13 needs to be considered.

The statement III) as it was originally stated appears to be wrong. In fact, in [2] for $q_4=1, q_5=2, q_6=3$ an isomorphism (defined over a field extension of $\mathbb{Q}$ of higher degree) was found between C and a curve C_k with $k=\mathbb{Q}$ (although it should be said that we don't have a precise proof of the fact that for these values of q_4, q_5, q_6 the curve C meets the condition $Aut(C)\cong\mathbb{Z}_2\oplus\mathbb{Z}_2\oplus\mathbb{Z}_2$).

Acknowledgements. We are grateful to Christophe Ritzenthaler who kindly pointed out the error to us and to our colleague Enrique González who carried out some computations in MAGMA for us.

References

[1] Y. Fuertes and G. González-Diez. *Fields of moduli and definition of hyperelliptic covers*, Arch. Math. 86 (2006) 398-408.

Key words and phrases. Hyperelliptic curves, automorphisms, field of moduli, field of definition.
2000 *Mathematics Subject Classification.* 14H37, 14H45, 14G99.

[2] R. Lercier, C. Ritzenthaler and J. Sijsling, *Fast computation of isomorphisms of hyperelliptic curves and explicit descent.* http://arxiv.org/abs/1203.5440

Departamento de Matemáticas,
U. Autónoma de Madrid,
28049 Madrid, Spain.
E-mail address: yolanda.fuertes@uam.es, gabino.gonzalez@uam.es