Theoretical challenge: **Periodically explicit**

Wavelets: theory and practice — Deadline: End of the course

The challenge

For $J \in \mathbb{Z}^+$ let f_J be the 1-periodic extension of

$$\sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} 2^{-j-1} \Big(\psi(2^{j+1}x - k) + \psi(-2^{j+1}x - k) \Big) \qquad \text{for} \quad |x| \le \frac{1}{2},$$

where ψ is the Haar wavelet. Give a simple fully explicit formula for the coefficients c_n^J in the Fourier expansion $\sum c_n^J e(nx)$ of f_J.

Comments

Let $c_n^\infty = \lim_{J\to+\infty} c_n^J$. With your formula you can check that although $b_n^J = c_n^J / c_n^\infty \to 1$ for each fixed n with $c_n^\infty \neq 0$, we have $b_{2^{J+1}-1}^J \to -\infty$. This is related to the high pitch that one hears sometimes when quantizing in time a pure tone (see the experimental challenge "step by step").