Non-mandatory problem 6

Deadline: December 1st

Let ω_1 and ω_2 be linearly independent (at each point) 1-forms on a manifold of dimension 2. Let A_1 and A_2 be functions such that $d\omega_1 = A_1 \ \omega_1 \wedge \omega_2$ and $d\omega_2 = A_2 \ \omega_1 \wedge \omega_2$, and define the 1-form $\theta = -A_1\omega_1 - A_2\omega_2$. Given a function f, consider now the same construction starting with $\widetilde{\omega}_1 = \cos f \ \omega_1 - \sin f \ \omega_2$ and $\widetilde{\omega}_2 = \sin f \ \omega_1 + \cos f \ \omega_2$ to get $\widetilde{\theta}$. Prove that $\widetilde{\theta} = \theta + df$.