Deadline: December 1st

Let $\omega_{1}$ and $\omega_{2}$ be linearly independent (at each point) 1-forms on a manifold of dimension 2 . Let $A_{1}$ and $A_{2}$ be functions such that $d \omega_{1}=A_{1} \omega_{1} \wedge \omega_{2}$ and $d \omega_{2}=A_{2} \omega_{1} \wedge \omega_{2}$, and define the 1-form $\theta=-A_{1} \omega_{1}-A_{2} \omega_{2}$. Given a function $f$, consider now the same construction starting with $\widetilde{\omega}_{1}=\cos f \omega_{1}-\sin f \omega_{2}$ and $\widetilde{\omega}_{2}=\sin f \omega_{1}+\cos f \omega_{2}$ to get $\widetilde{\theta}$. Prove that $\widetilde{\theta}=\theta+d f$.

