Deadline: November 13th

Prove, directly, that $d\omega$ is well-defined, i.e, that it does not depend on the choice of the coordinate chart.

Here "directly" means that we can only use calculus and the very definition:
Recall that for

$$\omega = \sum_{i_1 < i_2 < \ldots < i_k} f_{i_1 i_2 \ldots i_k}\, dx^{i_1} \wedge dx^{i_2} \wedge \cdots \wedge dx^{i_k}.$$

we define

$$d\omega = \sum_{j} \sum_{i_1 < i_2 < \ldots < i_k} \frac{\partial f_{i_1 i_2 \ldots i_k}}{\partial x^j}\, dx^j \wedge dx^{i_1} \wedge dx^{i_2} \wedge \cdots \wedge dx^{i_k}$$

If it is of any help, you can assume that the underlying manifold is $\mathbb{R}^n$.