

Deadline: October 14th

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Prove, directly, without using calculus of variations, that the 1-dimensional Euler-Lagrange equations are invariant by an arbitrary change of coordinates.

In other words, if  $L = L(x, \dot{x})$ , you have to prove that

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) = \frac{\partial L}{\partial x} \quad \Rightarrow \quad \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}} \right) = \frac{\partial L}{\partial y}$$

for a change of variables  $y = y(x)$ .

I emphasize that the proof has to be direct, using the chain rule and basic calculus a number of times. For instance,

$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial y} \frac{\partial y}{\partial x} + \frac{\partial L}{\partial \dot{y}} \frac{\partial \dot{y}}{\partial x}.$$

Note that in this expression, we would usually write  $y'(x)$  instead of  $\partial y / \partial x$ .