Agostinho Neto University | College of Sciences

Master in Mathematics and Applications

Test 2: Wavelets and signal processing

21 / 07 / 2021

NAME: ____

N. 1	N. 2	N. 3	TOTAL

---

1. (7 points) Let $J = [-2, -\frac{3}{2}] \cup [\frac{3}{2}, 2]$.

(a) Give the definition of the Fourier transform $\mathcal{F}(f)$ of a function f in $L^1(\mathbb{R})$ and compute $\mathcal{F}(\chi_J)$, where χ_J denotes the characteristic function of J.

(b) Prove that the collection $\left\{e^{-2\pi ikx}\chi_J\right\}_{k\in\mathbb{Z}}$ is and orthonormal system for $L^2(J)$.

---

2. (6 points)

(a) Give the definition of the Discrete Fourier Transform (DFT) of a discrete signal f of size N.

(b) Let $g = (g(0), g(1), g(2), g(3)) = (1, -1, 1, -1)$ be a discrete signal of size 4. Compute the Discrete Fourier Transform (DFT) of g.

---

3. (7 points)

For $k = 1, 2, \ldots, N$ and $n = 0, 1, \ldots, N-1$ define

$$S_k(n) = \mu_k \sqrt{\frac{2}{N}} \sin \frac{k\pi}{N}(n + \frac{1}{2}),$$

where $\mu_k = 1$ if $k = 1, 2, \ldots, N-1$ and $\mu_N = 1/\sqrt{2}$. Let $S_k = (S_k(n))_{n=0}^{N-1}$ be a discrete signal of size N.

(a) Show that $\|S_N\| = 1$ and that $\|S_k\| = 1$ for all $k = 1, 2, \ldots, N-1$.

(b) Prove that if $1 \le k < \ell \le N$ then $\langle S_k, S_\ell \rangle = 0$.

TIME: 90 minutes