Universidad Autónoma de Madrid
Master in Mathematics and Applications
Wavelets and Signal Processing - 2015-16 — Homework 4

Due: Wednesday, April 20, 2016

Coding and Entropy.

1. Show that for any source of information X with N symbols

$$0 \leq \mathcal{E}(X) \leq \log_2 N,$$

where $\mathcal{E}(X) = \sum_{k=1}^{n} p_k \log_2 \frac{1}{p_k}$ is the Shannon entropy of X for the probabilities $p_k, k = 0, 1, 2, \ldots, N$. (Hint: Use Lagrange multipliers to show that the maxima of $\mathcal{E}(X)$ in the region $0 \leq p_k \leq 1, k = 1, 2, \ldots, N$, with the restriction $\sum_{k=1}^{N} p_k = 1$ is attained at $p_k = 1/N, k = 1, 2, \ldots, N$.

2. Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ be a source of information whose probabilities are

$$p_1 = 0.49,\ p_2 = 0.26,\ p_3 = 0.12, p_3 = 0.04,\ p_5 = 0.04,\ p_6 = 0.03, p_7 = 0.02.$$

a) Compute the entropy of X. Build a binary Huffman code $\mathcal{C}$ for these probabilities and compute $\mathcal{A}_{\mathcal{C}}(X)$.

b) Suppose that the symbols of X are codify with a ternary code that takes values 0, 1, and 2. The code can now be represented in a ternary tree. Extend Huffman algorithm to find a ternary tree for X having the prefix condition and compute the average length of its codified words.

3. The quantize coefficients of an 8×8 block of an image are

$$\begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -4 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a binary Huffman code to represent these symbols.