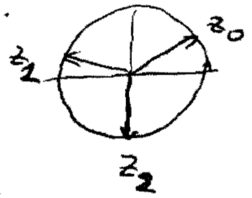
1º QUÍMICAS / MATEMÁTICAS / 27-11-2013 / 1er PARCIAL

1. a) $z = i = 1e^{i\pi/2}$; $z_k = 1\, e^{i\left(\frac{\pi/2 + 2k\pi}{3}\right)}$, $k = 0, 1, 2.$



$\underline{k=0}$: $z_0 = e^{\frac{\pi}{6} i} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2} i$

$\underline{k=1}$: $z_1 = e^{i\left(\frac{\pi}{6} + \frac{2\pi}{3}\right)} = \cos 150^\circ + i\sin 150^\circ = -\frac{\sqrt{3}}{2} + \frac{1}{2} i$

$\underline{k=2}$: $z_2 = e^{i\left(\frac{\pi}{6} + \frac{4\pi}{3}\right)} = \cos 270^\circ + i\sin 270^\circ = -i$

——— x ———

b) Candidatos $x = 3$, $x = -3$

$\boxed{x=3}$ $\lim\limits_{x\to 3^+} \frac{x^3+4x^2+4x+3}{x^2-9} = \frac{27+36+12+3}{0} = \infty$

$\lim\limits_{x\to 3^-} \frac{x^3+4x^2+4x+3}{x^2-9} = -\infty$

$x = 3$ es una asíntota vertical

$\boxed{x=-3}$ $\lim\limits_{x\to -3} \frac{x^3+4x^2+4x+3}{x^2-9} = \frac{-27+36-12+3}{9-9} = \frac{0}{0}$ (L'Hôpital)

$= \lim\limits_{x\to -3} \frac{3x^2+8x+4}{2x} = \frac{27-24+4}{-6} = -\frac{7}{6}$

$\boxed{x=-3}$ No es una asíntota vertical

——— x ———

c) $\mathrm{Dom}(f) = (0,\infty)$, $\mathrm{Rec}(f) = (-\infty,\infty)$

$x = 2 + \ln(3y) \iff x-2 = \ln(3y) \iff 3y = e^{x-2} \Rightarrow y = \frac{1}{3} e^{x-2}$

$f^{-1}(x) = \frac{1}{3} e^{x-2}$ con $\mathrm{Dom}(f^{-1}) = (-\infty,\infty)$, $\mathrm{Rec}(f^{-1}) = (0,\infty)$

——— x ———

d)

$\frac{d\, f\circ g}{dx} = f'(g(x))\cdot g'(x) \Rightarrow \left[e^{g(x)^2} + \ln(g(x)+1)\right](1+\cos^2(x))$

$(f\circ g)'(0) = (e^1 + \ln(2))(1+\cos 0) = 2(e + \ln 2)$

——— x ———

2 a) $v(t) = N'(t) = \dfrac{(1+\frac{t}{10})^2 - 2t(1+\frac{t}{10})\cdot\frac{1}{10}}{(1+\frac{t}{10})^4} = \dfrac{1+\frac{t}{10}-\frac{t}{5}}{(1+\frac{t}{10})^3} = \dfrac{1-\frac{t}{10}}{(1+\frac{t}{10})^3} = 0$

$\Rightarrow \boxed{t=10}$

t	0		10	
$v(t)$		+ crece	|	− decrece

La población de insectos crece entre 0 y 10 días y decrece a partir del décimo día.

b) Tenemos que hallar el máximo de $v(t) = \dfrac{1-\frac{t}{10}}{(1+\frac{t}{10})^3}$.

$$v'(t) = \frac{-\frac{1}{10}(1+\frac{t}{10})^3 - (1-\frac{t}{10})\,3(1+\frac{t}{10})^2\cdot\frac{1}{10}}{(1+\frac{t}{10})^6} = \frac{-\frac{1}{10}(1+\frac{t}{10}) - \frac{3}{10}(1-\frac{t}{10})}{(1+\frac{t}{10})^4}$$

$$= \frac{-\frac{1}{10}-\frac{t}{100}-\frac{3}{10}+\frac{3t}{100}}{(1+\frac{t}{10})^4} = \frac{-\frac{4}{10}+\frac{2t}{100}}{(1+\frac{t}{10})^4} = 0 \quad \Rightarrow$$

$$-\frac{4}{10}+\frac{2t}{100} = 0 \Rightarrow 2t = \frac{4}{10}\times 100 \Rightarrow \boxed{t=20.}$$

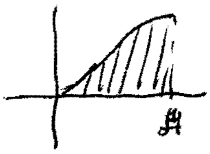
	0		20	
$v'(t)$		−	Mínimo	+

Comparamos:

t	0	20	∞
$v(t)$	1	$\frac{-1}{27}$	$\lim_{t\to\infty} v(t) = 0$

Luego la velocidad de crecimiento máxima para esta población de insectos se alcanza cuando $t=0$ y su valor es $v(0)=1$.

———— x ————

3 a) 

$$A = \int_0^{\pi} \operatorname{sen}^2\left(\frac{x}{2}\right) dx = \int_0^{\pi} \frac{1-\cos x}{2}\, dx$$

$$= \left[\frac{1}{2}x - \frac{\operatorname{sen} x}{2}\right]_0^{\pi} = \frac{\pi}{2}$$

b) $$V = \pi \int_0^{\pi} \operatorname{sen}^4\left(\frac{x}{2}\right) dx = \pi \int_0^{\pi} \left(\frac{1-\cos x}{2}\right)^2 dx = \pi \int_0^{\pi} \frac{1-2\cos x + \cos^2 x}{4}\, dx$$

$$= \pi \left[\frac{1}{4}x - \frac{1}{2}\operatorname{sen} x\right]_0^{\pi} + \pi \frac{1}{4} \int_0^{\pi} \frac{1+\cos 2x}{2}\, dx =$$

$$= \pi \left[\frac{\pi}{4} - 0\right] + \frac{\pi}{4}\left[\frac{1}{2}x + \frac{\operatorname{sen} 2x}{4}\right]_0^{\pi} = \frac{\pi^2}{4} + \frac{\pi^2}{8} = \frac{3\pi^2}{8}$$