

$$d = 8$$

$$\Psi_{\mathbb{Q}}(8) \supseteq \left\{ \begin{array}{c} (3), (5), (6), (8), (10), (12), (15), (16), (17), (20), (21), (24), (30), (32), \\ (2, 4), (2, 6), (2, 8), (2, 10), (2, 12), (2, 16), (2, 20), (2, 24), (3, 3), (3, 6), (3, 12), (4, 4), (4, 8), (4, 12), (5, 5), (6, 6) \end{array} \right\}$$

$G$	$\Psi_{\mathbb{Q}}(8, G) \setminus \{G\} \supseteq$
()	{(3), (5), (15), (17), (3, 3), (5, 5)}
(2)	{(6), (8), (10), (12), (16), (20), (24), (32), (2, 4), (2, 6), (2, 8), (2, 10), (2, 12), (2, 16), (2, 20), (2, 24), (3, 6), (3, 12), (4, 4), (4, 8), (4, 12), (6, 6)}
(3)	{(15)}
(4)	{(12), (16), (20), (24), (32), (2, 8), (2, 16), (2, 20), (2, 24), (4, 4), (4, 8), (4, 12)}
(5)	{(15)}
(6)	{(24), (2, 12), (2, 24), (3, 12), (4, 12)}
(7)	{(21)}
(8)	{(24), (32), (2, 16), (4, 8)}
(9)	{}
(10)	{(30), (2, 20)}
(12)	{(2, 24), (4, 12)}
(2, 2)	{(2, 6), (2, 8), (2, 10), (2, 12), (2, 16), (2, 20), (2, 24), (4, 4), (4, 8), (4, 12)}
(2, 4)	{(2, 12), (2, 16), (4, 8)}
(2, 6)	{(2, 24), (4, 12)}
(2, 8)	{(2, 24)}

$$g_{\mathbb{Q}}(8) \geq 9 \quad \#\mathcal{G}_{\mathbb{Q}}(8) \geq 200 \quad M_{\mathbb{Q}}(8) \geq 223494$$

$G$	$\mathcal{H}_{\mathbb{Q}}(8, E)$	Label
()	(3)	11a2
()	(3, 3)	338d2
()	(5)	225a2
()	(15)	50a2
()	(3), (5)	361a1
()	(3), (5, 5)	99d2
()	(3), (17)	14450n1
()	(5), (3, 3)	121b1
()	(15), (3, 3)	338d1
(2)	(4, 12)	36a3
(2)	(2, 4), (2, 6)	223494n1
(2)	(2, 4), (2, 12)	36a4
(2)	(2, 12) <sup>2</sup>	80b1
(2)	(6), (2, 4)	200b2
(2)	(6), (4, 4)	128a1
(2)	(6), (4, 8)	200c4
(2)	(12), (4, 4)	576a1
(2)	(2, 4) <sup>2</sup> , (2, 6)	2205a2
(2)	(2, 4) <sup>2</sup> , (6, 6)	726a1
(2)	(6), (2, 4) <sup>2</sup>	46a1
(2)	(6), (2, 4), (2, 8)	200b1
(2)	(6), (2, 4), (2, 20)	450a4
(2)	(6), (2, 8), (4, 4)	49a1
(2)	(6), (2, 10), (4, 8)	64a4
(2)	(6), (2, 20) <sup>2</sup>	450a3
(2)	(6), (4, 8) <sup>2</sup>	6150n6
(2)	(6), (8) <sup>2</sup>	320a4
(2)	(6), (10), (4, 4)	2304h1
(2)	(6), (16), (4, 8)	24a6
(2)	(6), (16) <sup>2</sup>	2880r6
(2)	(8), (2, 4), (2, 24)	960o3
(2)	(8), (2, 12), (2, 24)	14400bo8
(2)	(8) <sup>2</sup> , (2, 12)	450g1
(2)	(8) <sup>2</sup> , (4, 12)	960o7
(2)	(8), (24), (2, 4)	30a3
(2)	(8), (24), (4, 12)	720j3
(2)	(12), (2, 4) <sup>2</sup>	14a3

$G$	$\mathcal{H}_{\mathbb{Q}}(8, E)$	Label
(2)	(12), (2, 4), (2, 12)	576a2
(2)	(12), (2, 12) <sup>2</sup>	98a3
(2)	(12), (2, 12), (4, 4)	130a4
(2)	(24), (2, 4), (2, 12)	1040g1
(2)	(24), (2, 4), (2, 24)	960e3
(2)	(2, 4), (2, 6) <sup>2</sup> , (3, 6)	256a2
(2)	(2, 4), (2, 12), (3, 12), (6, 6)	98a4
(2)	(6), (2, 4) <sup>2</sup> , (2, 10)	66c3
(2)	(6), (2, 8) <sup>2</sup> , (4, 4)	17a3
(2)	(6), (2, 16) <sup>2</sup> , (4, 8)	75b6
(2)	(6), (8), (2, 4) <sup>2</sup>	128a2
(2)	(6), (8), (2, 4), (2, 8)	33a2
(2)	(6), (8), (2, 4), (2, 16)	63a1
(2)	(6), (8) <sup>2</sup> , (2, 4)	120b1
(2)	(6), (8) <sup>2</sup> , (4, 4)	33a4
(2)	(6), (8), (16), (2, 4)	126b1
(2)	(6), (8), (32), (2, 4)	3150bk1
(2)	(6), (10), (2, 4) <sup>2</sup>	576f2
(2)	(6), (10), (2, 8), (4, 4)	256b1
(2)	(6), (16), (2, 8), (4, 8)	42a5
(2)	(6), (16) <sup>2</sup> , (4, 8)	48a4
(2)	(6), (20), (2, 4) <sup>2</sup>	150b3
(2)	(8), (2, 4), (2, 8), (2, 12)	14400bo1
(2)	(8), (2, 8), (2, 12) <sup>2</sup>	14400ef8
(2)	(8) <sup>2</sup> , (2, 4), (2, 12)	210a3
(2)	(8) <sup>2</sup> , (2, 12) <sup>2</sup>	30a8
(2)	(8), (12), (2, 4) <sup>2</sup>	130a3
(2)	(8), (24), (2, 4), (2, 12)	30a7
(2)	(10), (2, 4) <sup>2</sup> , (2, 6)	288a2
(2)	(10), (12), (2, 4) <sup>2</sup>	288a1
(2)	(12), (2, 4), (2, 12), (6, 6)	3136k3
(2)	(24), (2, 4), (2, 8), (2, 12)	14400ef7
(2)	(24), (2, 8), (2, 12) <sup>2</sup>	14400bo7
(2)	(2, 4) <sup>2</sup> , (2, 6) <sup>2</sup> , (3, 6)	1210a1
(2)	(2, 4), (2, 6) <sup>2</sup> , (2, 8), (3, 6)	256a1
(2)	(6), (2, 8) <sup>3</sup> , (4, 4)	21a6
(2)	(6), (8) <sup>2</sup> , (2, 4) <sup>2</sup>	49a2

$G$	$\mathcal{H}_{\mathbb{Q}}(8, E)$	Label
(2)	$(6), (8)^2, (2, 8), (4, 4)$	294f2
(2)	$(6), (8)^2, (2, 10), (4, 4)$	576h1
(2)	$(6), (8)^2, (10), (4, 4)$	256b2
(2)	$(6), (8), (10), (2, 4), (2, 8)$	288d2
(2)	$(6), (8), (16), (2, 4), (2, 8)$	126b3
(2)	$(6), (8), (16), (2, 4), (2, 16)$	45a1
(2)	$(6), (8), (32), (2, 4), (2, 16)$	75b1
(2)	$(6), (16), (2, 8)^2, (4, 4)$	21a5
(2)	$(6), (16)^3, (4, 8)$	4800cd6
(2)	$(6), (20), (2, 4)^2, (2, 10)$	1014c1
(2)	$(6), (20), (2, 8)^2, (4, 4)$	32a3
(2)	$(8)^2, (2, 12)^2, (4, 4)$	90c4
(2)	$(8)^2, (12), (2, 10), (4, 4)$	288d4
(2)	$(8), (10), (12), (2, 4), (2, 8)$	288d3
(2)	$(8), (24), (2, 4), (2, 8), (2, 12)$	90c5
(2)	$(8), (24), (2, 8), (2, 12)^2$	1050k4
(2)	$(8), (24), (2, 12)^2, (4, 4)$	960e7
(2)	$(12)^2, (2, 4)^2, (6, 6)$	112c3
(2)	$(24)^2, (2, 4), (2, 8), (2, 12)$	720j7
(2)	$(24)^2, (2, 8), (2, 12)^2$	150c8
(2)	$(24)^2, (2, 12)^2, (4, 4)$	150c7
(2)	$(6), (8), (16)^2, (2, 4), (2, 8)$	45a7
(2)	$(6), (16), (2, 8)^3, (4, 4)$	48a2
(2)	$(6), (16)^2, (2, 8)^2, (4, 4)$	40a2
(2)	$(12), (2, 4)^2, (2, 6)^2, (3, 6)$	726a2
(2)	$(6), (8), (16)^3, (2, 4), (2, 8)$	45a3
(2)	$(6), (16)^2, (2, 8)^3, (4, 4)$	75b7
(2)	$(6), (16)^3, (2, 8)^2, (4, 4)$	195a7
(2)	$(6), (16)^3, (2, 8)^3, (4, 4)$	15a5
(2)	$(6), (16)^4, (2, 8)^2, (4, 4)$	15a6
(2)	$(6), (16)^4, (2, 8)^3, (4, 4)$	210e7
(3)	(15)	225b1
(4)	$(12), (4, 8)$	2880be6
(4)	$(2, 8), (2, 24), (4, 4)$	960o8
(4)	$(2, 24)^2, (4, 12)$	150c3
(4)	$(12), (2, 8)^2$	17a1
(4)	$(12), (2, 8), (4, 4)$	17a4

$G$	$\mathcal{H}_{\mathbb{Q}}(8, E)$	Label
(4)	(12), (2, 16), (4, 8)	1470k1
(4)	(12), (2, 20), (4, 8)	32a1
(4)	(12), (4, 8) <sup>2</sup>	40a4
(4)	(12), (16), (4, 8)	72a1
(4)	(12), (2, 8) <sup>2</sup> , (4, 4)	33a3
(4)	(12), (2, 8), (2, 16), (4, 4)	144b4
(4)	(12), (2, 16) <sup>2</sup> , (4, 8)	192c6
(4)	(12), (16), (2, 8), (4, 4)	63a5
(4)	(12), (16) <sup>2</sup> , (4, 8)	24a4
(4)	(12), (20), (2, 8), (4, 4)	32a4
(4)	(24), (2, 8) <sup>2</sup> , (4, 4)	90c1
(4)	(24), (2, 8) <sup>2</sup> , (4, 12)	720j8
(4)	(24), (2, 8), (2, 24), (4, 4)	1050k1
(4)	(12), (16), (2, 8), (2, 16), (4, 4)	21a4
(4)	(12), (16), (2, 16) <sup>2</sup> , (4, 8)	294c1
(4)	(12), (16) <sup>2</sup> , (2, 8), (4, 4)	40a3
(4)	(12), (16) <sup>2</sup> , (2, 16) <sup>2</sup>	240d6
(4)	(12), (16) <sup>2</sup> , (4, 8) <sup>2</sup>	510e6
(4)	(12), (16) <sup>2</sup> , (2, 16) <sup>2</sup> , (4, 8)	1680p6
(4)	(12), (16), (32), (2, 8), (2, 16), (4, 4)	15a7
(5)	(15)	11a1
(6)	(4, 12)	36a1
(6)	(2, 12) <sup>2</sup>	14a4
(6)	(2, 12) <sup>2</sup> , (3, 12)	14a1
(6)	(24), (2, 12) <sup>2</sup>	130a1
(6)	(24), (2, 12), (2, 24)	90c8
(6)	(24) <sup>2</sup> , (2, 12)	30a1
(6)	(24) <sup>2</sup> , (4, 12)	90c7
(7)	(21)	26b1
(8)	(24), (2, 16) <sup>2</sup>	15a4
(8)	(24), (2, 16), (4, 8)	1230f1
(8)	(24), (2, 16) <sup>2</sup> , (4, 8)	21a3
(8)	(24), (32) <sup>2</sup> , (2, 16), (4, 8)	210e1
(10)	(30), (2, 20) <sup>2</sup>	66c1
(12)	(2, 24) <sup>2</sup> , (4, 12)	90c3
(2, 2)	(2, 6)	120b2
(2, 2)	(4, 12)	14400bo6

$G$	$\mathcal{H}_{\mathbb{Q}}(8, E)$	Label
(2, 2)	(2, 6), (2, 8)	289a2
(2, 2)	(2, 6), (4, 4)	153c2
(2, 2)	(2, 6), (4, 8)	441c2
(2, 2)	(2, 8), (4, 12)	150c6
(2, 2)	(2, 12) <sup>2</sup>	30a6
(2, 2)	(2, 24), (4, 12)	960o6
(2, 2)	(2, 6), (2, 8) <sup>2</sup>	17a2
(2, 2)	(2, 6), (2, 8), (4, 4)	33a1
(2, 2)	(2, 6), (2, 8), (4, 8)	40a1
(2, 2)	(2, 6), (2, 10), (4, 4)	288d1
(2, 2)	(2, 6), (2, 16), (4, 8)	45a2
(2, 2)	(2, 6), (4, 8) <sup>2</sup>	72a2
(2, 2)	(2, 12) <sup>3</sup>	210a6
(2, 2)	(2, 6), (2, 8) <sup>3</sup>	1287e4
(2, 2)	(2, 6), (2, 8) <sup>2</sup> , (2, 16)	75b3
(2, 2)	(2, 6), (2, 8) <sup>2</sup> , (4, 4)	45a5
(2, 2)	(2, 6), (2, 8) <sup>2</sup> , (4, 8)	63a4
(2, 2)	(2, 6), (2, 8), (2, 10), (4, 8)	64a1
(2, 2)	(2, 6), (2, 8), (2, 16) <sup>2</sup>	1470k2
(2, 2)	(2, 6), (2, 8), (2, 16), (4, 4)	585f2
(2, 2)	(2, 6), (2, 8), (2, 16), (4, 8)	225c2
(2, 2)	(2, 6), (2, 8), (2, 20), (4, 8)	32a2
(2, 2)	(2, 6), (2, 8), (4, 8) <sup>2</sup>	24a2
(2, 2)	(2, 6), (2, 16) <sup>2</sup> , (4, 8)	630c2
(2, 2)	(2, 6), (4, 8) <sup>3</sup>	15a2
(2, 2)	(2, 8), (2, 12) <sup>2</sup> , (4, 4)	90c2
(2, 2)	(2, 12) <sup>3</sup> , (4, 4)	14400bo2
(2, 2)	(2, 6), (2, 8) <sup>4</sup>	231a3
(2, 2)	(2, 6), (2, 8) <sup>3</sup> , (4, 4)	306c3
(2, 2)	(2, 6), (2, 8) <sup>3</sup> , (4, 8)	21a2
(2, 2)	(2, 6), (2, 8) <sup>2</sup> , (2, 16) <sup>2</sup>	1680p2
(2, 2)	(2, 6), (2, 8) <sup>2</sup> , (2, 16), (4, 4)	225c5
(2, 2)	(2, 6), (2, 8) <sup>2</sup> , (2, 16), (4, 8)	294c4
(2, 2)	(2, 6), (2, 8) <sup>2</sup> , (4, 8) <sup>2</sup>	45a4
(2, 2)	(2, 6), (2, 8), (2, 16) <sup>2</sup> , (4, 8)	4800cd5
(2, 2)	(2, 6), (2, 8) <sup>4</sup> , (4, 4)	441c3
(2, 2)	(2, 6), (2, 8) <sup>4</sup> , (4, 8)	975a3

$G$	$\mathcal{H}_{\mathbb{Q}}(8, E)$	Label
(2, 2)	(2, 6), (2, 8) <sup>3</sup> , (2, 16), (4, 4)	1470k5
(2, 2)	(2, 6), (2, 8) <sup>2</sup> , (2, 16) <sup>2</sup> , (4, 8)	240d2
(2, 4)	(2, 12)	231a2
(2, 4)	(2, 12), (4, 8)	120a2
(2, 4)	(2, 12), (2, 16), (4, 8)	15a3
(2, 4)	(2, 12), (4, 8) <sup>2</sup>	24a1
(2, 4)	(2, 12), (2, 16) <sup>3</sup>	1230f2
(2, 4)	(2, 12), (2, 16) <sup>2</sup> , (4, 8)	195a2
(2, 4)	(2, 12), (2, 16), (4, 8) <sup>2</sup>	15a1
(2, 4)	(2, 12), (4, 8) <sup>3</sup>	195a3
(2, 4)	(2, 12), (2, 16) <sup>3</sup> , (4, 8)	510e2
(2, 4)	(2, 12), (2, 16) <sup>2</sup> , (4, 8) <sup>2</sup>	210e3
(2, 4)	(2, 12), (2, 16) <sup>2</sup> , (4, 8) <sup>3</sup>	240d5
(2, 6)	(2, 24), (4, 12)	90c6
(2, 8)	(2, 24)	210e2