Hoja 5

1) The number 2^{29} has 9 distinct digits. Find which digit is missing.

2) Prove that for every n, there exist n consecutive integers each of which is divisible by at least two different primes.

3) Let $n \geq m \geq 1$ be two non-negative integers and let $d = \gcd(m, n)$. Prove that $\frac{d}{n}\binom{n}{m}$ is an integer.

4) Let $a_1 = 3$, and for $n \geq 1$ let $a_{n+1} = 3^{a_n}$. Which integers between 00 and 99 inclusive occur as the last two digits in the decimal expansion of infinitely many a_n? *Hint: if a is coprime to n then* $a^b \pmod{n} = a^{b \pmod{\phi(n)}} \pmod{n}$.

5) Show that for every prime p there is an integer n such that $2^n + 3^n + 6^n - 1$ is divisible by p.

6) Let n be an even positive integer. Prove that $n^2 - 1$ divides $2^{n!} - 1$.