Hoja 3

1) Let $c_0 > 0$, $c_1 > 0$, and $c_{n+1} = \sqrt{c_n} + \sqrt{c_{n-1}}$ for $n = 1, 2, \ldots$. Prove that the sequence $\{c_n\}$ has a limit and find this limit.

2) The set $\mathbb{N}$ of natural numbers is partitioned into two infinite subsets, A and B. Prove that for any real $c > 0$ there exist increasing sequences $\{a_n\}$ and $\{b_n\}$ such that $a_n \in A$, $b_n \in B$, and $\text{lím}_{n\to\infty} \frac{a_n}{b_n} = c$.

3) Does there exist a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x)$ is irrational for x rational and is rational for x irrational?

4) Let $n > 1$ be an odd integer. Prove that n does not divide $3^n + 1$.

5) Determine all rational solutions of the equation $x^3 + y^3 = x^2 + y^2$.

6) Let p be a prime greater than 5. Prove that $p - 4$ cannot be the fourth power of an integer.