Hoja 2

1) Let $a_1 \geq a_2 \geq \ldots \geq a_n$ and $b_1 \geq b_2 \geq \ldots \geq b_n$ positive real numbers. Prove that

$$\prod_{k=1}^{n} (a_k + b_k) \leq \sqrt[n]{\prod_{i,h=1}^{n} (a_i + b_j)}.$$

2) Find all the solutions to the equation

$$1! + 2! + 3! + \ldots + n! = m^2,$$

for m, n positive integers.

3) Prove that the sequence $1, 11, 111, \ldots$ contains an infinite subsequence whose terms are pairwise relatively prime.

4) The sequence $\{x_n\}$ of positive real numbers decreases and satisfies $\sum_{n=1}^{\infty} x_n = \infty$. Prove that

$$\sum_{n=1}^{\infty} x_n \exp\left(-\frac{x_n}{x_{n+1}}\right) = \infty$$

5) Prove that the expression

$$\frac{\gcd(m,n)}{n} \binom{n}{m}$$

is an integer for all pairs of integers $n \geq m \geq 1$.

6) A real sequence $\{x_n\}_{n\geq 1}$ satisfies the inequalities $|x_n - x_m| > \frac{1}{n}$ for any $n < m$. Prove that this sequence is unbounded.