Hoja 1

1) Prove that

$$(1+1/2)(1+1/4)\cdot\ldots\cdot(1+1/2^n) < 3,$$

for every natural number n.

2) Let a, b, c be non-negative real numbers. Show that

$$(a+b)(b+c)(c+a) \geq 8abc.$$

3) Prove that any two numbers of the following sequence are relatively prime:

$$2+1,\ 2^2+1,\ 2^4+1,\ \ldots, 2^{2^n}+1.$$

Remark: The result obtained here proves that there is an infinite number of primes.

4) Prove that any positive rational number r can be represented as a finite sum of rationals of the form $1/k$ with distinct denominators.

5) Does there exist a continuous function f on $\mathbb{R}$ such that $f(f(x)) = e^{-x}$, $x \in \mathbb{R}$?

6) Denote by $[x]$ the integer part of x. Prove that for any natural number n, the integer $[(2+\sqrt{3})^n]$ is odd.