Hoja 10

1) **(a)** Find all numbers n between 1 and 100 such that $(n-1)!$ is not divisible by n;

(b) Find all numbers n between 1 and 100 such that $(n-1)!$ is not divisible by n^2.

2) Let $a_1, a_2, a_3, b_1, b_2, b_3$ be positive real numbers. Prove that

$$\sqrt[3]{(a_1 + b_1)(a_2 + b_2)(a_3 + b_3)} \geq \sqrt[3]{a_1 a_2 a_3} + \sqrt[3]{b_1 b_2 b_3}.$$

3) Let G be a connected graph with k edges. Show that it is possible to label the edges of this graph with the numbers $1, 2, \ldots, k$, so that for every vertex that belongs to at least two edges, the greatest common divisor of the integers that label the edges containing this vertex is equal to 1.

4) Given 2^{n-1} subsets of a set with n elements with the property that any three of them have a nonempty intersection, prove that the intersection of all of these sets is nonempty.