Hoja 2

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1) There are n identical cars on a circular track. Among all of them, they have just enough gas for one car to complete a lap. Show that there is a car which can complete a lap by collecting gas from the other cars on its way around.

2) Each of the numbers $a_1, a_2, \ldots, a_n$ is 1 or -1, and we have

$$a_1a_2a_3a_4 + a_2a_3a_4a_5 + \cdots + a_na_1a_2a_3 = 0.$$

Prove that n is a multiple of 4.

3) Find all the integer solutions of the equation

$$x^4 + y^4 = 3x^3y.$$

4) Let $x, y, z > 1$. Prove that

$$\frac{x^4}{(y-1)^2} + \frac{y^4}{(z-1)^2} + \frac{z^4}{(x-1)^2} \geq 48.$$

5) Let a and b be two rational numbers such that $\sqrt[3]{a} + \sqrt[3]{b}$ is also rational. Assuming that $a > 0$, $b > 0$, prove that both $\sqrt[3]{a}$ and $\sqrt[3]{b}$ are rational numbers.

6) Let N_n denote the number of ordered n-tuples of positive integers $(a_1, a_2, \ldots, a_n)$ such that $1/a_1 + 1/a_2 + \cdots + 1/a_n = 1$. Determine whether N_{10} is even or odd.