1) Prove that every nonnegative integer can be represented in the form \( a^2 + b^2 - c^2 \), where \( a, b \) and \( c \) are positive integers with \( a < b < c \).

2) Prove that there does not exist a non-constant polynomial with integral coefficients such that \( P(0), P(1), P(2), \ldots \) are all prime numbers.

3) Consider the polynomial \( f(X) := X^2 - 4 \), with \( a \in \mathbb{Z} \). Let \( f^{(n)} = f \circ \cdots \circ f \) be its \( n \)th iterate and let \( c_n = f^{(n)}(0) \). Prove that if \( p \) is an odd prime such that \( p \mid (c_n + 2) \) for some \( n \), then \( p \mid (c_m - 2) \) for all \( m \).

4) Prove that the equation \( p^2 + q^2 = r^2 + s^2 + t^2 \) has no solution with primes \( p, q, r, s, t \).

5) Call a real-valued function \( f \) very convex if

\[
\frac{f(x) + f(y)}{2} \geq f\left(\frac{x + y}{2}\right) + |x - y|
\]

holds for all real numbers \( x \) and \( y \). Prove that no very convex function exists.

6) Let \( P_n \) be the set of subsets of \( \{1, 2, \ldots, n\} \). Let \( c(n, m) \) be the number of functions \( f : P_n \to \{1, 2, \ldots, m\} \) such that \( f(A \cap B) = \min\{f(A), f(B)\} \). Prove that

\[
c(n, m) = \sum_{j=1}^{m} j^n.
\]

7) Given a function \( f : [0, 1] \to \mathbb{R} \), we associate with it the set

\[
E(f) = \{x \in [0, 1] : \lim_{t \to x} f(t) = +\infty\}.
\]

Is it true that for any \( f \), \( E(f) \) is (at most) countable?

8) Let \( a_1, a_2, \ldots, a_n \) be positive numbers and let \( s \) be their sum. Prove that

\[
(1 + a_1)(1 + a_2)\ldots(1 + a_n) \leq 1 + s + \frac{s^2}{2!} + \frac{s^3}{3!} + \cdots + \frac{s^n}{n!}.
\]

9) Show that for any natural number \( n \), there exist infinitely many positive Fibonacci numbers divisible by \( n \).