Problem 1. Find all positive functions $f : (0, +\infty) \to (0, +\infty)$ such that
$$f(f(x)) = 3x - f(x), \quad \forall x > 0.$$ 

Problem 2. Does there exist a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f(x)$ is irrational for any rational $x$ and $f(x)$ is rational for any irrational $x$?

Problem 3. Given an odd prime $p$, denote by $\mathbb{Z}/p\mathbb{Z}$ the set of residues modulo $p$ (it is a field). How many elements has the set
$$\{x^2 : x \in \mathbb{Z}/p\mathbb{Z}\} \cap \{y^2 + 1 : y \in \mathbb{Z}/p\mathbb{Z}\}.$$

Problem 4. Suppose that functions $f$ and $g$ are defined on $\mathbb{R}$, are periodic (possibly with different periods) and satisfy
$$\lim_{x \to +\infty} f(x) - g(x) = 0.$$ 

Prove that $f(x) \equiv g(x)$. 