Problem 1. Consider a doubly infinite table of positive integers:

\[
\begin{array}{cccc}
  a_{1,1} & a_{1,2} & a_{1,3} & \cdots \\
  a_{2,1} & a_{2,2} & a_{2,3} & \cdots \\
  a_{3,1} & a_{3,2} & a_{3,3} & \cdots \\
  \vdots & \vdots & \vdots & \ddots \\
\end{array}
\]

Suppose that each positive integer appear exactly 10 times in this table. Prove that for some \( m \) and \( n \), \( a_{m,n} > mn \).

Problem 2. Suppose that a given function \( p(x) \) increases on \([0, c]\) and is differentiable on \((0, c]\), whereas \( p'(x) \) decreases on \((0, c]\). Prove that

\[
\sum_{n=1}^{\infty} \frac{1}{n^2} p\left(\frac{c}{n}\right) < +\infty.
\]

Problem 3. A ball moves endlessly on a circular billiard table. When it hits the edge it is reflected. Show that if it passes through a point on the table three times, then it passes through it infinitely many times.

The same question for a billiard with an elliptic shape.

Problem 4. Determine all integers \( n > 2 \) with the property that there exists one of the numbers \( 1, 2, \ldots, n + 1 \) such that after its removal, the \( n \) numbers left can be arranged as \( a_1, a_2, \ldots, a_n \) with no two of \(|a_1 - a_2|, |a_2 - a_3|, \ldots, |a_{n-1} - a_n|, |a_n - a_1| \) being equal.

Problem 5. The sequence \( \{x_n\} \) is defined by \( x_1 = 1/2 \) and \( x_{n+1} = x_n - x_n^2 \) for \( n \geq 1 \). Prove that

\[
\lim_{n \to \infty} nx_n = 1.
\]

Problem 6. One starts with three line segments of lengths 1, 2, 3. Then the segment of length 3 is cut into \( n \geq 2 \) line segments. Prove that from these \( n + 2 \) segments, three segments can be chosen that can form a triangle.