Hoja 1

Problem 1. Find all prime numbers p such that $2p^4 - p^2 + 16$ is a perfect square.

Problem 2. Does there exist a function $f : \mathbb{R} \to \mathbb{R}$ such that for arbitrary real numbers x, y, we have

$$f(x - f(y)) \leq x - yf(x)?$$

Problem 3. $2n$ real numbers with a positive sum are placed on a circle. Each of these $2n$ numbers defines two sets of n consecutive numbers with this number at the end. Prove that for at least one of the given $2n$ numbers, both of the two sets it defines has a positive sum.

Problem 4. A sequence of numbers is defined recursively by $u_1 = 2$, $u_2 = 8$, $u_n = 4u_{n-1} - u_{n-2}$, $n = 3, 4, 5, \ldots$.

Prove that

$$\frac{\pi}{12} = \sum_{n=1}^{\infty} \operatorname{arccot} u_n^2.$$