Hoja 9

1) Let \( f : [0, 1] \rightarrow \mathbb{R} \) be a Riemann integrable function that satisfies the functional equation

\[
f(x) = \frac{1}{3} \left( f\left( \frac{x}{3} \right) + f\left( \frac{x+1}{3} \right) + f\left( \frac{x+2}{3} \right) \right)
\]

for all \( x \in [0, 1] \). Determine the function \( f \), if it is known that \( f(1/\pi) = 1 \).

2) Let \( P(x) \) be a polynomial of degree \( n \), all whose roots are real and distinct, and let \( c \) be a positive number. The set of real numbers \( x \) such that \( P'(x)/P(x) > c \) is a union of finitely many disjoint intervals. Prove that the sum of their lengths equals \( n/c \).

3) Does there exist an injective function \( f : \mathbb{R} \rightarrow \mathbb{R} \) that attains a maximal value on any non-empty subset of \( \mathbb{R} \)?

4) Find all possible integral solution to the following equation:

\[
19y^2 = 20x^3 - 2019
\]

5) Does there exist a \( 12 \times 12 \) matrix \( A \), all whose entries are numbers 0, \pm 1, such that \( \det A = 2018 \)?