Hoja 9

1) Let $f : [0, 1] \to \mathbb{R}$ be a Riemann integrable function that satisfies the functional equation

$$f(x) = \frac{1}{3}\Big(f\big(\frac{x}{3}\big) + f\big(\frac{x+1}{3}\big) + f\big(\frac{x+2}{3}\big)\Big)$$

for all $x \in [0, 1]$. Determine the function f, if it is known that $f(1/\pi) = 1$.

2) Let $P(x)$ be a polynomial of degree n, all whose roots are real and distinct, and let c be a positive number. The set of real numbers x such that $P'(x)/P(x) > c$ is a union of finitely many disjoint intervals. Prove that the sum of their lengths equals n/c.

3) Does there exist an injective function $f : \mathbb{R} \to \mathbb{R}$ that attains a maximal value on any non-empty subset of $\mathbb{R}$?

4) Find all possible integral solution to the following equation:

$$19y^2 = 20x^3 - 2019$$

5) Does there exist a 12×12 matrix A, all whose entries are numbers $0, \pm 1$, such that $\det A = 2018$?