1) Find all functions \( f : \mathbb{R} \rightarrow \mathbb{R} \) satisfying the functional equation
\[
f(f(x) + y) = 2x + f(f(y) - x).
\]

2) Let \( f \) be a continuous strictly increasing function on \((0, +\infty)\), satisfying
\[
\lim_{x \to +\infty} \frac{f(x)}{x \log x} = 1.
\]
Denote by \( \varphi(x) \) the function, inverse to \( f \) (that is, it satisfies \( \varphi(f(x)) = x \) for any positive \( x \)). Prove that
\[
\lim_{x \to +\infty} \frac{\varphi(x)}{x / \log x} = 1.
\]

3) Count the number of \( 3 \times 3 \) matrices whose entries belong to \(-1, 0, 1\) and whose determinant is of the form \( 3k + 1, k \in \mathbb{Z} \).

4) a) Does there exist a continuous function \( f \) on \( \mathbb{R} \) such that \( f(f(x)) = e^{-x} \), for all \( x \in \mathbb{R} \).
   b) The same question for the functional equation \( f(f(x)) = e^x \), for all \( x \in \mathbb{R} \).

5) In a real square matrix, all elements except the diagonal ones are given. Prove that one can place zeros and ones on the diagonal so that the resulting matrix is invertible.