Hoja 3

1) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. A real number y is called *an extremal value* of f if there exists a point x_0 such that $f(x_0) = y$ and x_0 is a point of local maximum or minimum of f. Prove that the set of extremal values of f is countable.

2) Prove that for any polynomial P with integer coefficients, there exists a positive integer k such that $P(k)$ is not prime.

3) Define a set to be self-referenced if its number of elements is its element. For instance, $\{1, 3, 4\}$ is self-referenced and $\{1, 2, 4\}$ is not. Find the number of subsets of $\{1, 2, \ldots, n\}$ that are minimal self-referenced sets, which means that they are self-referenced and do not have any smaller self-referenced subset.

4) Show that there is an infinite number of powers of two which start by the digit 9.

5) Which of the two polynomials,

$$P(x) = (1 + x^2 - x^3)^{1000}, \quad Q(x) = (1 - x^2 + x^3)^{1000},$$

has larger coefficient at x^{400}?