Hoja 2

Algebra

1) Suppose that a finite group has exactly n elements of order p, where p is a prime. Prove that either $n = 0$ or p divides $n + 1$.

Analysis

2) Let $x_1, \ldots, x_n$ be positive numbers. Prove that

$$\big(\max_{1\le j\le n} x_j\big)(x_1 + 2x_2 + \cdots + nx_n) \ge \frac{1}{2}(x_1 + x_2 + \cdots + x_n)^2.$$

---

3) Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that for any real numbers $a < b$, the image $f([a, b])$ is a closed interval of length $b - a$.

---

4) Prove that the squares with sides $1/1$, $1/2$, $1/3, \ldots$ may be placed into the square with side $3/2$ so that no two of them will have any interior point in common.

Combinatorics

5) The alphabet of a certain language has three letters, and the words are formed using this alphabet. Some words of two or more letters are prohibited, and any two prohibited words are of different lengths. Show that one can form a word of arbitrary length that does not contain any forbidden word.