Hoja 1

1) Let $f : [0,1] \to [0,1]$ be a non-decreasing function. Is it true that there exists $x \in [0,1]$ such that $f(x) = x$? Is the same thing true if f is non-increasing?

2) Suppose $A \subset \{1, 2, 3, \ldots, 2n\}$ is any set with $n + 1$ elements.

a) Show that there are always two numbers in A which are coprime.

b) Show that there are always two numbers in A such that one divides the other.

3) A magician has one hundred cards numbered from 1 to 100. He distributes them between three boxes (a red one, a white one, and a blue one), so that each box contains at least one card. A person from the audience draws two cards from two different boxes and announces the sum of the numbers on those cards. With this information, the magician is able to locate the box from which no card has been drawn. How many ways are there to distribute the cards between the three boxes so that the trick will work?

4) Let r be an arbitrary positive rational number. Prove that it can be represented as a sum of fractions of the form $\frac{1}{k}$, where all denominators are different.

5) Prove that a convex 13-gon (that is, a polygon with 13 sides) cannot be cut into finitely many parallelograms.