

UAM Lectures on PDEs and Geometry

# Parabolic equations with rough coefficients and singular forcing

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We are interested in parabolic differential equations  $(\partial_t - a\partial_x^2)u = f$  with a very irregular forcing  $f$  and only mildly regular coefficients  $a$ . This is motivated by stochastic differential equations, where  $f$  is random, and quasilinear equations, where  $a$  is a (nonlinear) function of  $u$ .

Below a certain threshold for the regularity of  $f$  and  $a$  (on the Hölder scale), giving even a sense to this equation requires a renormalization. In the framework of the above setting, we present recent ideas from the area of stochastic differential equations (Lyons' rough path, Gubinelli's controlled rough paths, Hairer's regularity structures) that allow to build a solution theory. We make a connection with Safonov's approach to Schauder theory.

This is based on joint work with H. Weber (and ongoing work with him, J. Sauer and S. Smith).