Stable solutions to some elliptic problems: minimal cones, the Allen-Cahn equation, and blow-up solutions

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Abstract:

We will present several results on the classification of stable solutions to some nonlinear elliptic equations. These results are a crucial step within the regularity theory of minimizers to such problems. We will mainly center our attention to three different (but connected) equations. Some techniques and ideas in the three settings are quite similar.

The first one is the celebrated result of Simons on the flatness of minimal cones in low dimensions, that we will describe in some detail. Its semilinear analogue is a conjecture on the Allen-Cahn equation posed by E. De Giorgi in 1978. This is our second problem, for which we will discuss some proofs, as well as an open problem (for high dimensions) on the saddle-shaped solution vanishing on the Simons cone.

The third problem concerns the boundedness of stable solutions to reaction-diffusion equations in bounded domains. We will present proofs on their regularity in low dimensions and discuss the main problem which remains still open. Finally, we will briefly comment on related results for harmonic maps, free boundary problems, as well as for nonlocal minimal cones and the fractional Allen-Cahn equation.