Winter workshop on elliptic and parabolic equations

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Organizers

Matteo Bonforte, María del Mar González, Ana Primo,
Fernando Quirós and Fernando Soria

Elvise Berchio, Politecnico di Torino, IT

Optimization of eigenvalues of partially hinged rectangular plates

ABSTRACT: We consider the spectrum of non-homogeneous partially hinged plates having structural engineering applications. A possible way to prevent instability phenomena is to maximize the ratio between the frequencies of certain oscillating modes with respect to the density function of the plate; we prove existence of optimal densities and we investigate their analytic expression. This analysis suggests where to locate reinforcing material within the plate; some numerical experiments give further information and support the theoretical results. Based on a joint work with Alessio Falocchi.

Manuel del Pino, University of Bath, UK

Singularities for the Keller-Segel System in $\mathbb{R}^2$

ABSTRACT: The Keller-Segel System in $\mathbb{R}^2$

$$\left\{ \begin{array}{ll} u_t = \Delta u - \nabla \cdot (u \nabla v) & \text{in } \mathbb{R}^2 \times (0, +\infty) \\ v = (-\Delta)^{-1} u := \frac{1}{2\pi} \int_{\mathbb{R}^2} \log \frac{1}{|x-z|} u(z,t) dz & \end{array} \right.$$

for $u > 0$, is the classical diffusion model for chemotaxis, the motion of a population of bacteria driven by standard diffusion and a nonlocal drift given by the gradient of a chemoattractant, a chemical the bacteria produce. It is well known that mass $M = \int_{\mathbb{R}^2} u(\cdot, t)$ is constant along this ow and that solutions blows-up in finite time or approach zero as $t \to \infty$ according to $M > 8\pi$ or $M < 8\pi$. The critical case $M = 8\pi$ is more delicate. Infinite-time blow-up or stabilization around a steady state depends on the second moment of the solution. We construct a solution globally defined in time that blows-up as $t \to +\infty$ with precise asymptotic profile, establishing stability of the phenomenon. The method applies to construct solutions that blow up in finite time simultaneously at several given points in the plane. We discuss the parallel of this phenomenon in the 2-dimensional harmonic map flow into the sphere S^2,

$$u_t = \Delta u + |\nabla u|^2 \qquad \text{in } \mathbb{R}^2 \times (0, T).$$

Serena Dipierro, University of Western Australia

A gradient estimate for nonlocal minimal graphs

ABSTRACT: In this talk we describe a minimization problem involving the biharmonic operator, and we discuss some results obtained in collaboration with Aram Karakhanyan and Enrico Valdinoci about the free boundary condition, the regularity of the solutions and that of their free boundary, and a monotonicity formula.

Veronica Felli, Università di Milano Bicocca, IT

On spectral stability of Aharonov-Bohm operators with moving poles

ABSTRACT: In this talk, I will present some results in collaboration with L. Abatangelo (Milano-Bicocca), L. Hillairet (Orlans), C. Lna (Stockholm), B. Noris (Amiens), and M. Nys, concerning the behavior of the eigenvalues of Aharonov-Bohm operators with one moving pole or two colliding poles. In both cases of poles moving inside the domain and approaching the boundary, the rate of the eigenvalue variation is estimated in terms of the vanishing order of some limit eigenfunction. An accurate blow-up analysis for scaled eigenfunctions will be presented too.

Xavier Fernández-Real, ETH, Zurich

The non-regular part of the free boundary for the fractional obstacle problem

ABSTRACT: The fractional obstacle problem in $\mathbb{R}^n$ with obstacle $\varphi \in C^\infty(\mathbb{R}^n)$ can be written as

$$\min\{(-\Delta)^s u, u - \varphi\} = 0, \quad \text{in} \quad \mathbb{R}^n.$$

The set $\{u = \varphi\} \subset \mathbb{R}^n$ is called the contact set, and its boundary is the free boundary, an unknown of the problem.

The free boundary for the fractional obstacle problem can be divided between two subsets: regular points (around which the free boundary is smooth, and is $n-1$ dimensional) and degenerate points.

The set of degenerate points, even for smooth obstacles, can be very large (for example, with infinite $\mathcal{H}^{n-1}$ measure). In a joint work with X. Ros-Oton we show, however, that generically solutions to the fractional obstacle problem have a lower dimensional degenerate set. That is, for almost every solution (in an appropriate sense), the set of degenerate points is lower dimensional.

Rafael López Soriano, Universidad de Valencia, ES

Existence and compactness for a problem arising in conformal geometry

ABSTRACT: This talk is concerned with a Liouville type problem on compact surfaces with boundary. More precisely, this equation allows us to assign Gauss and geodesic curvatures under a conformal change of the metric. We derive existence using the variational structure of the problem and compactness of solutions analyzing the blow-up phenomenon. Joint works with L. Battaglia (Roma Tre), A. Malchiodi (SNS Pisa) and D. Ruiz (Granada).

Monica Musso, University of Bath, UK

Gluing methods for vortices in Euler equations

ABSTRACT: The Euler equations (1755) define a system of non-linear PDEs that models the dynamics of an inviscid, incompressible fluid. In dimension 2, a classical problem is the desingularized N-vortex problem, namely the existence of true smooth solutions of Euler equations with highly concentrated vorticities around N points. Using gluing methods, for any sufficiently small $\varepsilon > 0$ we show the existence of a solution with vorticity of the form $\omega_\varepsilon(x,t) \approx \sum_{j=1}^{N} \frac{\kappa_j}{\varepsilon^2} W(\frac{x-\xi_j(t)}{\varepsilon})$ where $W(y) = \frac{8}{(1+|y|^2)^2}$ is the standard Liouville bubble which has mass 8π, $\kappa_j \in \mathbb{R}$ and the centers $\xi_j(t)$ solve the Kirchoff-Routh law of motion. This refines a previous construction by Marchioro and Pulvirenti, giving precise description of the velocity field.

In dimension 3,if the initial vorticity is concentrated along a smooth curve in space, a long standing question is whether the associated solution exhibits a vorticity still very concentrated around a curve on finite times. The formal derivation of the motion of the curve was first computed by Da Rios in 1903, and it approximately evolves by the bi-normal flow of curves. Jerrard and Seis used refined energy estimates to prove the validity of the asymptotic law under the assumption that vorticity is indeed concentrated at all time. The big open problem is whether one can find solutions of the Euler equations for which the vorticity remains close for a significant period of time to a filament evolving by binormal flow. We prove that this is the case when the curve is an helix evolving by bi-normal flow. Using helical symmetries and looking for rotating solutions, this problem can be reduced to finding a concentrating solution for a 2-dimensional elliptic problem in divergence form. We also show that, given a curve evolving by bi-normal flow, for any k there exist an approximate solution for which the largest term in the error is of size $O(\frac{\varepsilon}{|\log \varepsilon|^k})$.

These results are in collaboration with Juan Dvila, Manuel del Pino and Juncheng Wei.

Yannick Sire, Johns Hopkins University, US

Geometric variational problems: old problems and new approaches

ABSTRACT: I will describe new methods to deal with geometric variational problem (like minimal surfaces or harmonic maps and their flows with free boundary) in relation with recent analytic results on nonlocal equations. It offers a fruitful new formalism and allows to use the theory of integro-differential equations developed in the last years to get new results. Emphasis will be given on GMT aspects and some of their surprising consequences in these contexts.

The talk will be accessible to a large audience and I will emphasize on developing some strategies for new directions of research and open problems in the field.

Diana Stan, Universidad de Cantabria, ES

Carleman estimates for fractional operators

ABSTRACT: In this talk I will present how to derive Carleman estimates for the fractional relativistic operator. Firstly, we consider changing-sign solutions to the heat equation for such operators. We prove monotonicity inequalities and convexity of certain energy functionals to deduce Carleman estimates with linear exponential weight. Our approach is based on spectral methods and functional calculus. Secondly, we use pseudo-differential calculus in order to prove Carleman estimates with quadratic exponential weight, both in parabolic and elliptic contexts. The latter also holds in the case of the fractional Laplacian.

Joint work with Luz Roncal and Luis Vega. Partially supported by the ERCEA Advanced Grant 2014 669689-HADE.

References:

L Roncal, D Stan, L Vega, Carleman type inequalities for fractional relativistic operators. preprint arXiv:1909.10065

Enrico Valdinoci, University of Western Australia

Nonlocal minimal graphs in the plane are generically sticky

ABSTRACT: We discuss some recent boundary regularity results for nonlocal minimal surfaces in the plane. In particular, we show that nonlocal minimal graphs in the plane exhibit generically stickiness effects and boundary discontinuities. More precisely, if a nonlocal minimal graph in a slab is continuous up to the boundary, then arbitrarily small perturbations of the far-away data necessarily produce boundary discontinuities. Hence, either a nonlocal minimal graph is discontinuous at the boundary, or a small perturbation of the prescribed conditions produces boundary discontinuities. The proof relies on a sliding method combined with a fine boundary regularity analysis, based on a discontinuity/smoothness alternative. Namely, we establish that nonlocal minimal graphs are either discontinuous at the boundary or their derivative is Hlder continuous up to the boundary. In this spirit, we prove that the boundary regularity of nonlocal minimal graphs in the plane "jumps" from discontinuous to differentiable, with no intermediate possibilities allowed. In particular, we deduce that the nonlocal curvature equation is always satisfied up to the boundary. As a byproduct of our analysis, one describes the "switch" between the regime of continuous (and hence differentiable) nonlocal minimal graphs to that of discontinuous (and hence with differentiable inverse) ones. These results have been obtained in collaboration with Serena Dipierro and Ovidiu Savin.

Zoran Vondraček, University of Zagreb

On the potential theory of Markov processes with jump kernels decaying at the boundary

ABSTRACT: Consider a β-stable process in the Euclidean space $\mathbb{R}^d$, $0 < \beta \leq 2$, which is killed upon exiting an open subset D. The killed process is then subordinated via an independent γ-stable subordinator. The resulting process Y^D is called a subordinate killed stable process. In two recent papers, it has been shown that the potential theory of this process exhibits some interesting features. The first one is the form of the jumping kernel which depends on the distance of points to the boundary

in a novel way. The second and unexpected feature is the fact that for some values of the stability index γ, the boundary Harnack principle fails.

In the first part of the talk, I will review these results. The second part of the talk will be devoted to ongoing work on potential theory of jump processes in open subset D of $\mathbb{R}^d$ defined through their jumping kernels $J^D(x,y)$ that depend not only on the distance between two points x and y, but also on the distance of each point to the boundary ∂D of the state space D. More precisely, I assume that $J^D(x,y) = B(x,y)|x-y|^{-d-\alpha}$, where the boundary term $B(x,y)$ is not necessarily bounded between two positive constants but is allowed to go to zero at the boundary. Under certain assumptions on $B(x,y)$, it is shown that the Harnack and Carleson inequalities are valid. I will also discuss the boundary Harnack principle in the case of the halfspace.

Joint work with Panki Kim and Renming Song.
