Basic Statistics and Probability (2018-19)
Science & Engineering Program Boston University-Faculty of Science UAM

Chapter 9: Inferences Based on Two Samples

The following problems are from McClave, J. and Sincich, T. (2017), *Statistics*, 13th. edition, Pearson. The number preceding the exercise is the corresponding one from this textbook. Some of the problems may also appear in previous editions of the book, possibly with different numbering.

9.6 In order to compare the means of two populations, independent random samples of 400 observations are selected from each population, with the following results:

Sample 1	Sample 2
$\bar{x}_1 = 5{,}275$	$\bar{x}_2 = 5{,}240$
$s_1 = 150$	$s_2 = 200$

a. Use a 95% confidence interval to estimate the difference between the population means $\mu_1 - \mu_2$. Interpret the confidence interval.
b. Test the null hypothesis $H_0 : (\mu_1 - \mu_2) = 0$ versus the alternative hypothesis $H_a : (\mu_1 - \mu_2) \neq 0$. Say as much as you can about the p-value of the test, and interpret the result.
c. Suppose the test in part **b** were conducted with the alternative hypothesis $H_a : (\mu_1 - \mu_2) > 0$. How would your answer to part **b** change?
d. Test the null hypothesis $H_0 : (\mu_1 - \mu_2) = 25$ versus the alternative $H_a : (\mu_1 - \mu_2) \neq 25$. Give the p-value, and interpret the result. Compare your answer with that obtained from the test conducted in part **b**.
e. What assumptions are necessary to ensure the validity of the inferential procedures applied in parts **a**–**d**?

9.9 Independent random samples from normal populations produced the following results:

Sample 1	Sample 2
1.2	4.2
3.1	2.7
1.7	3.6
2.8	3.9
3.0	

a. Calculate the pooled estimate of σ^2.
b. Do the data provide sufficient evidence to indicate that $\mu_2 > \mu_1$? Testusing $\alpha = .10$.
c. Find a 90% confidence interval for $(\mu_1 - \mu_2)$.

9.16 Cognitive impairment of schizophrenics. A study of the differences in cognitive function between normal individuals and patients diagnosed with schizophrenia was published in the *American Journal of Psychiatry* (Apr. 2010). The total time (in minutes) a subject spent on the Trail Making Test (a standard psychological test) was used as a measure of cognitive function. The researchers theorize that the mean time on the Trail Making Test for schizophrenics will be larger than the corresponding mean for normal subjects. The data for independent random samples of 41 schizophrenics and 49 normal individuals yielded the following results:

	Schizophrenia	Normal
Sample size	41	49
Mean time	104.23	62.24
Standard deviation	45.45	16.34

a. Define the parameter of interest to the researchers.
b. Set up the null and alternative hypothesis for testing the researchers' theory.
c. The researchers conducted the test, part **b**, and reported a p-value of .001. What conclusions can you draw from this result? (Use $\alpha = .01$.)
d. Find a 99% confidence interval for the target parameter. Interpret the result.

9.19 Bulimia study. The "fear of negative evaluation" (FNE) scores for 11 female students known to suffer from the eating disorder bulimia and 14 female students with normal eating habits are reproduced in the next table. (Recall that the higher the score, the greater is the fear of a negative evaluation.)

Bulimic students:	21	13	10	20	25	19	16	21	24	13	14			
Normal students:	13	6	16	13	8	19	23	18	11	19	7	10	15	20

a. Locate a 95% confidence interval for the difference between the population means of the FNE scores for bulimic and normal female students on the MINITAB printout below. Interpret the result.

MINITAB Output for Exercise 9.19

Two-Sample T-Test and CI: FNESCORE, GROUP

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Two-sample T for FNESCORE

GROUP     N   Mean  StDev  SE Mean
Bulimic  11  17.82   4.92      1.5
Normal   14  14.14   5.29      1.4


Difference = mu (Bulimic) - mu (Normal)
Estimate for difference:  3.68
95% CI for difference:  (-0.60, 7.95)
T-Test of difference = 0 (vs not =): T-Value = 1.78  P-Value = 0.089  DF = 23
Both use Pooled StDev = 5.1303
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b. What assumptions are required for the interval of part **a** to be statistically valid? Are these assumptions reasonably satisfied? Explain.

9.45 The placebo effect and pain. According to research published in *Science* (Feb. 20, 2004), the mere belief that you are receiving an effective treatment for pain can reduce the pain you actually feel. Researchers tested this placebo effect on 24 volunteers as follows: Each volunteer was put inside a magnetic resonance imaging (MRI) machine for two consecutive sessions. During the first session, electric shocks were applied to their arms and the blood oxygen leveldependent (BOLD) signal (a measure related to neural activity in the brain) was recorded during pain. The second session was identical to the first, except that, prior to applying the electric shocks, the researchers smeared a cream on the volunteer's arms. The volunteers were informed that the cream would block the pain when, in fact, it was just a regular skin lotion (i.e., a placebo). If the placebo is effective in reducing the pain experience, the BOLD measurements should be higher, on average, in the first MRI session than in the second.

a. Identify the target parameter for this study.
b. What type of design was used to collect the data?
c. Give the null and alternative hypotheses for testing the placebo effect theory.
d. The differences between the BOLD measurements in the first and second sessions were computed and summarized in the study as follows: $n_d = 24$, $\bar{x}_d = .21$, $s_d = .47$. Use this information to calculate the test statistic.
e. The p-value of the test was reported as p-value = .02. Make the appropriate conclusion at $\alpha = .05$.

9.63 Bullying behavior study. School bullying is a form of aggressive behavior that occurs when a student is exposed repeatedly to negative actions (e.g., name-calling, hitting, kicking, spreading slander) from another student. In order to study the effectiveness of an antibullying policy at Dutch elementary schools, a survey of over 2,000 elementary school children was conducted (*Health Education Research*, Feb. 2005). Each student was asked if he or she ever bullied another student. In a sample of 1,358 boys, 746 claimed they had never bullied another student. In a sample of 1,379 girls, 967 claimed they had never bullied another student.

a. Estimate the true proportion of Dutch boys who have never bullied another student.
b. Estimate the true proportion of Dutch girls who have never bullied another student.
c. Estimate the difference in the proportions with a 90% confidence interval.
d. Make a statement about how likely the interval you used in part **c** contains the true difference in proportions.
e. Which group is more likely to bully another student, Dutch boys or Dutch girls?

9.68 Planning-habits survey. *American Demographics* (Jan. 2002) reported the results of a survey on the planning habits of men and women. In response to the question "What is your preferred method of planning and keeping track of meetings, appointments, and deadlines?" 56% of the men and 46% of the women answered "I keep them in my head." A nationally representative sample of 1,000 adults participated in the survey; therefore, assume that 500 were men and 500 were women.

a. Set up the null and alternative hypotheses for testing whether the percentage of men who prefer keeping track of appointments in their head is larger than the corresponding percentage of women.
b. Compute the test statistic for the test.
c. Give the rejection region for the test, using $\alpha = .01$.
d. Find the p-value for the test.
e. Draw the appropriate conclusion.

9.87 Bulimia study. Refer to the *American Statistician* (May 2001) study comparing the "fear of negative evaluation" (FNE) scores for bulimic and normal female students. Suppose you want to estimate $(\mu_B - \mu_N)$, the difference between the population means of the FNE socres for bulimic and normal female students, using a 95% confidence interval with a sampling error of two points. Find the sample sizes required to obtain such an estimate. Assume that from previous pilot samples we have $s_B^2 = s_N^2 = 25$.

9.94 Rat damage in sugarcane. Poisons are used to prevent rat damage in sugarcane fields. An agriculture department is investigating whether the rat poison should be located in the middle of the field or on the outer perimeter. One way to answer this question is to determine where the greater amount of damage occurs. If damage is measured by the proportion of cane stalks that have been damaged by rats, how many stalks from each section of the field should be sampled in order to estimate the true difference between proportions of stalks damaged in the two sections, to within .03 with 90% confidence?