Basic Statistics and Probability (2018-19)
Science & Engineering Program Boston University-Faculty of Science UAM

Chapter 8: Inferences Based on a Single Sample. Tests of Hypothesis

The following problems are from McClave, J. and Sincich, T. (2017), *Statistics*, 13th. edition, Pearson. The number preceding the exercise is the corresponding one from this textbook. Some of the problems may also appear in previous editions of the book, possibly with different numbering.

8.13 Play Golf America program. In the Play Golf America program, teaching professionals at participating golf clubs provide a free 10-minute lesson to new customers. According to the Professional Golf Association (PGA), golf facilities that participate in the program gain, on average, $2,400 in green fees, lessons, or equipment expenditures. A teaching professional at a golf club believes that the average gain in green fees, lessons, or equipment expenditures for participating golf facilities exceeds $2,400.

a. In order to support the claim made by the teaching professional, what null and alternative hypotheses should you test?
b. Suppose you select $\alpha = .05$. Interpret this value in the words of the problem.
c. For $\alpha = .05$, specify the rejection region of a large-sample test.

8.16 Calories in school lunches. India's Mid-Day Meal scheme mandates that high schools that are part of this scheme must serve lunches that contain at least 700 calories and 20 grams of protein. Suppose a nutritionist believes that the true mean number of calories served at lunch at all high schools that are part of this scheme is less than 700 calories.

a. Identify the parameter of interest.
b. Specify the null and alternative hypotheses for testing this claim.
c. Describe a Type I error in the words of the problem.
d. Describe a Type II error in the words of the problem.

8.40 Heart rate during laughter. Laughter is often called "the best medicine," since studies have shown that laughter can reduce muscle tension and increase oxygenation of the blood. In the *International Journal of Obesity* (Jan. 2007), researchers at Vanderbilt University investigated the physiological changes that accompany laughter. Ninety subjects (18–34 years old) watched film clips designed to evoke laughter. During the laughing period, the researchers measured the heart rate (beats per minute) of each subject, with the following summary results: $\bar{x} = 73.5$, $s = 6$. It is well known that the mean resting heart rate of adults is 71 beats per minute.

a. Set up H_0 and H_a for testing whether the true mean heart rate during laughter exceeds 71 beats per minute.
b. If $\alpha = .05$, find the rejection region for the test.
c. Calculate the value of the test statistic.
d. Make the appropriate conclusion.

8.45 Bone fossil study. Archeologists have found that humerus bones from the same species of animal tend to have approximately the same length–to–width ratios. It is known that species A exhibits a mean ratio of 8.5. Suppose 41 fossils of humerus bones were unearthed at an archeological site in East Africa, where species A is believed to have lived. (Assume that the unearthed bones were all from the same unknown species.) The length–to–width ratios of the bones are listed in the following table.

10.73	8.89	9.07	9.20	10.33	9.98	9.84	9.59
8.48	8.71	9.57	9.29	9.94	8.07	8.37	6.85
8.52	8.87	6.23	9.41	6.66	9.35	8.86	9.93
8.91	11.77	10.48	10.39	9.39	9.17	9.89	8.17
8.93	8.80	10.02	8.38	11.67	8.30	9.17	12.00
9.38							

To analyze the data with R we write:

```
X = c(10.73,8.89,9.07,9.20,10.33,9.98,9.84,9.59,
      8.48,8.71,9.57,9.29,9.94,8.07,8.37,6.85,
      8.52,8.87,6.23,9.41,6.66,9.35,8.86,9.93,
      8.91,11.77,10.48,10.39,9.39,9.17,9.89,8.17,
      8.93,8.80,10.02,8.38,11.67,8.30,9.17,12.00,9.38)
mean(X)
[1] 9.257561
sd(X)
[1] 1.203565
```

a. Test whether the population mean ratio of all bones of this particular species differs from 8.5. Use $\alpha = .01$.
b. What are the practical implications of the test you conducted in part **a.**?

8.62 Crab spiders hiding on flowers. Refer to the *Behavioral Ecology* (Jan. 2005) experiment on crab spiders' use of camouflage to hide from predators (e.g., birds) on flowers, presented in Exercise 2.42. Researchers at the French Museum of Natural History collected a sample of 10 adult female crab spiders, each sitting on the yellow central part of a daisy, and measured the chromatic contrast between each spider and the flower. The data (for which higher values indicate a greater contrast, and, presumably, an easier detection by predators) are shown in the accompanying table. The researchers discovered that a contrast of 70 or greater allows birds to see the spider. Of interest is whether the true mean chromatic contrast of crab spiders on daisies is less than 70.

57	75	116	37	96	61	56	2	43	32

Based on Thery, M., et al. "Specific color sensitivities of prey and predator explain camouflage in different visual systems." *Behavioral Ecology*, Vol. 16, No. 1, Jan. 2005 (Table 1).

a. Define the parameter of interest, μ.
b. Set up the null and alternative hypotheses of interest.
c. Find $\bar{x}$ and s for the sample data, and then use these values to compute the test statistic.
d. Give the rejection region for $\alpha = .10$.
e. State the appropriate conclusion in the words of the problem.
f. When analyzing the data with R we obtain the following result. Interpret the p-value.

```
X = c(57,75,116,37,96,61,56,2,43,32)
t.test(X,mu=70,alternative="less")

        One Sample t-test

data:  X
t = -1.2112, df = 9, p-value = 0.1283
alternative hypothesis: true mean is less than 70
95 percent confidence interval:
     -Inf 76.41899
sample estimates:
mean of x
     57.5
```

8.80 Paying for music downloads. If you use the Internet, have you ever paid to access or download music? This was one of the questions of interest in a recent *Pew Internet and American Life Project Survey* (Oct. 2010). In a representative sample of 755 adults who use the Internet, 506 stated that they have paid to download music. Let p represent the true proportion of all Internet-using adults who have paid to download music.

a. Compute a point estimate of p.
b. Set up the null and alternative hypotheses for testing whether the true proportion of all Internet-using adults who have paid to download music exceeds .7.
c. Compute the test statistic for part b.
d. Find the rejection region for the test if $\alpha = .01$.
f. Make the appropriate conclusion using the rejection region.
g. Make the appropriate conclusion using the p-value obtained in the following R output:

```
prop.test(506,755,p=0.7,alternative="greater",correct=FALSE)

        1-sample proportions test without continuity correction

data:  506 out of 755, null probability 0.7
X-squared = 3.193, df = 1, p-value = 0.963
alternative hypothesis: true p is greater than 0.7
95 percent confidence interval:
 0.6414909 1.0000000
sample estimates:
        p
0.6701987
```

8.150 Errors in medical tests. Medical tests have been developed to detect many serious diseases. A medical test is designed to minimize the probability that it will produce a “false positive” or a “false negative.” A false positive is a positive test result for an individual who does not have the disease, whereas a false negative is a negative test result for an individual who does have the disease.

a. If we treat a medical test for a disease as a statistical test of hypothesis, what are the null and alternative hypotheses for the medical test?
b. What are the Type I and Type II errors for the test? Relate each to false positives and false negatives.

8.160 Federal civil trial appeals. A breakdown of 678 civil cases that were originally tried in front of a judge (in federal civil trials) and appealed by either the plaintiff or the defendant is reproduced in the table

Outcome of Appeal	Number of Cases
Plaintiff trial win—reversed	71
Plaintiff trial win—affirmed/dismissed	240
Defendant trial win—reversed	68
Defendant trial win—affirmed/dismissed	299
Total	678

Do the data provide sufficient evidence to indicate that the percentage of civil cases appealed that are actually reversed is less than 25%? Test, using $\alpha = .01$.

8.163 Radioactive lichen. Lichen has a high absorbance capacity for radiation fallout from nuclear accidents. University of Alaska researchers collected nine lichen specimens at various locations and measured the amount of the radioactive element cesium–137 (in microcuries per milliliter) in each specimen. The data values, converted to logarithms, are given in the following table:

Location			
Bethel	−5.50	−5.00	
Eagle Summit	−4.15	−4.85	
Moose Pass	−6.05		
Turnagain Pass	−5.00		
Wickersham Dome	−4.10	−4.50	−4.60

Based on Lichen Radionuclide Baseline Research Project, 2003, p. 25. *Orion*, University of Alaska–Fairbanks.

Assume that in previous years the mean cesium amount in lichen was $\mu = .003$ microcurie per milliliter. Is there sufficient evidence to indicate that the mean amount of cesium in lichen specimens differs from this value?