Chapter 8: One-sample hypothesis tests

Radioactive lichen. Lichen has a high absorbance capacity for radiation fallout from nuclear accidents. University of Alaska researchers collected nine lichen specimens at various locations and measured the amount of the radioactive element cesium–137 (in microcuries per milliliter) in each specimen. The data values, converted to logarithms are given in the following table:

Location			
Bethel	−5.50	−5.00	
Eagle Summit	−4.15	−4.85	
Moose Pass	−6.05		
Turnagain Pass	−5.00		
Wickersham Dome	−4.10	−4.50	−4.60

Based on Lichen Radionuclide Baseline Research Project, 2003, p. 25.
***Orion*, University of Alaska–Fairbanks.**

Assume that in previous years the mean cesium amount in lichen was $\mu = .003$ microcurie per milliliter. Is there sufficient evidence to indicate that the mean amount of cesium in lichen specimens differs from this value?

X = amount of cesium-137 in one lichen specimen chosen at random

$E(X) = \mu$ = mean amount of cesium-137 in lichen specimens

We want to test

$H_0: \mu = 0.003$ $\quad \alpha$ (we'll take $\alpha = 0.05$)

$H_1: \mu \neq 0.003$

Since $n=9$ is too small a sample size to use the large-sample approximation given by the CLT, we have to assume that X follows a normal distribution: $X \sim N(\mu, \sigma)$. Then the rejection region for the test is $R = \{|t| > t_{8;\alpha/2}\}$, where the test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

To compute $\bar{x}$, we have to take into account that, for example, $-5.50 = \log(x_1) \Leftrightarrow x_1 = e^{-5.50} = 0.004$. So the original cesium readings were

$x_1 = 0.004$, $x_2 = e^{-5.00} = 0.007$, $x_3 = e^{-4.15} = 0.016$, $x_4 = e^{-4.85} = 0.008$

$x_5 = e^{-6.05} = 0.002$, $x_6 = e^{-5.00} = 0.007$, $x_7 = e^{-4.10} = 0.017$,

$x_8 = e^{-4.50} = 0.011$, $x_9 = e^{-4.60} = 0.010$

$\bar{x} = 0.009 \quad s = 0.005 \longrightarrow t = \dfrac{0.009 - 0.003}{0.005/\sqrt{9}} = 3.72 > t_{8;0.05} = 1.860 \Rightarrow$

$\Rightarrow$ We reject $H_0: \mu = 0.003$ at the 5% significance level.