6.15

X is a discrete random variable with possible values 2, 4, 9

Consider the following probability distribution:

x	2	4	9
$p(x)$	1/3	1/3	1/3

a. Calculate μ for this distribution.
b. Find the sampling distribution of the sample mean $\bar{x}$ for a random sample of $n = 3$ measurements from this distribution, ~~and show that $\bar{x}$ is an unbiased estimator of μ.~~
c. Find the sampling distribution of the sample median M for a random sample of $n = 3$ measurements from this distribution, and show that the median is a biased estimator of μ.
d. If you wanted to use a sample of three measurements from this population to estimate μ, which estimator would you use? Why?

a. $E(X) = \sum_x x \cdot p(x) = 2 \cdot \frac{1}{3} + 4 \cdot \frac{1}{3} + 9 \cdot \frac{1}{3} = \frac{1}{3}(2+4+9) = 5$

b.

Sample draws (n=3)			Sample mean	Sample median
x_1	x_2	x_3	$\bar{x}$	M
2	2	2	2	2
2	2	4	8/3	2
2	2	9	13/3	2
2	4	2	8/3	2
2	4	4	10/3	4
2	4	9	5	4
2	9	2	13/3	2
2	9	4	5	4
2	9	9	20/3	9
4	2	2	8/3	2
4	2	4	10/3	4
4	2	9	5	4
4	4	2	10/3	4
4	4	4	4	4
4	4	9	17/3	4
4	9	2	5	4
4	9	4	17/3	4
4	9	9	22/3	9
9	2	2	13/3	2
9	2	4	5	4
9	2	9	20/3	9
9	4	2	5	4
9	4	4	17/3	4
9	4	9	22/3	9
9	9	2	20/3	9
9	9	4	22/3	9
9	9	9	9	9

Since 2, 4 and 9 are equally likely, the probability of obtaining any of the 27 sample draws on the left is $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}$.

The sampling distribution of the sample mean $\bar{x}$ consists in giving the possible values of $\bar{x}$ and the probability of their appearance:

Possible values of $\bar{x}$	Probability
2	1/27
8/3	3/27 = 1/9
13/3	3/27 = 1/9
10/3	3/27 = 1/9
5	6/27 = 2/9
20/3	3/27 = 1/9
4	1/27
17/3	3/27 = 1/9
22/3	3/27 = 1/9
9	1/27
Total	1

c. Sampling distribution of the median M:

Possible values of M	Probability
2	$7/27$
4	$12/27$
9	$7/27$