Basic Statistics and Probability (2018-19)
Science & Engineering Program Boston University-Faculty of Science UAM

Chapter 6: Sampling Distributions

The following problems are from McClave, J. and Sincich, T. (2017), *Statistics*, 13th. edition, Pearson. The number preceding the exercise is the corresponding one from this textbook. Some of the problems may also appear in previous editions of the book, possibly with different numbering.

6.3 The probability distribution shown here describes a population of measurements that can assume values of 0, 2, 4, and 6, each of which occurs with the same relative frequency:

x	0	2	4	6
$p(x)$	1/4	1/4	1/4	1/4

a. List all the different samples of $n = 2$ measurements that can be selected from this population.
b. Calculate the mean of each different sample listed in part **a.**.
c. If a sample of $n = 2$ measurements is randomly selected from the population, what is the probability that a specific sample will be selected?
d. Assume that a random sample of $n = 2$ measurements is selected from the population. List the different values of $\bar{x}$ found in part **b.**, and find the probability of each. Then give the sampling distribution of the sample mean $\bar{x}$ in tabular form.
e. Construct a probability histogram for the sampling distribution of $\bar{x}$.

6.14 Consider the following probability distribution:

x	0	1	4
$p(x)$	1/3	1/3	1/3

a. Find μ and σ^2.
b. Find the sampling distribution of the sample mean $\bar{x}$ for a random sample of $n = 2$ measurements from this distribution.
c. Show that $\bar{x}$ is an unbiased estimator of μ. [Hint: Show that $E(\bar{x}) = \sum \bar{x}p(\bar{x}) = \mu$.]
d. Find the sampling distribution of the sample variance s^2 for a random sample of $n = 2$ measurements from this distribution.
e. Show that s^2 is an unbiased estimator for σ^2.

6.15 Consider the following probability distribution:

x	2	4	9
$p(x)$	1/3	1/3	1/3

a. Calculate μ for this distribution.
b. Find the sampling distribution of the sample mean $\bar{x}$ for a random sample of $n = 3$ measurements from this distribution, and show that $\bar{x}$ is an unbiased estimator of μ.
c. Find the sampling distribution of the sample median M for a random sample of $n = 3$ measurements from this distribution, and show that the median is a biased estimator of μ.
d. If you wanted to use a sample of three measurements from this population to estimate μ, which estimator would you use? Why?

6.27 Suppose a random sample of n measurements is selected from a population with mean $\mu = 100$ and variance $\sigma^2 = 100$. For each of the following values of n, give the mean and standard deviation of the sampling distribution of the sample mean $\bar{x}$.

a. $n = 4$
b. $n = 25$
c. $n = 100$
d. $n = 50$
e. $n = 500$
f. $n = 1000$

6.28 Suppose a random sample of $n = 25$ measurements is selected from a population with mean μ and standard deviation σ. For each of the following values of μ and σ, give the values of $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$.

a. $\mu = 10$, $\sigma = 3$
b. $\mu = 100$, $\sigma = 25$
c. $\mu = 20$, $\sigma = 40$
d. $\mu = 10$, $\sigma = 100$

6.30 A random sample of $n = 64$ observations is drawn from a population with a mean equal to 20 and standard deviation equal to 16.

a. Give the mean and standard deviation of the (repeated) sampling distribution of $\bar{x}$.
b. Describe the shape of the sampling distribution of $\bar{x}$. Does your answer depend on the sample size?
c. Calculate the standard normal z-score corresponding to a value of $\bar{x} = 16$.
d. Calculate the standard normal z-score corresponding to a value of $\bar{x} = 23$.
e. Find $P(\bar{x} < 16)$.
f. Find $P(\bar{x} > 23)$.
g. Find $P(16 < \bar{x} < 23)$.

6.38 Cost of unleaded fuel. According to the American Automobile Association (AAA), the average cost of a gallon of regular unleaded fuel at gas stations in May 2014 was \$3.65 (*AAA Fuel Gauge Report*). Assume that the standard deviation of such costs is \$.15. Suppose that a random sample of $n = 100$ gas stations is selected from the population and the cost per gallon of regular unleaded fuel is determined for each. Consider $\bar{x}$, the sample mean cost per gallon.

a. Calculate $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$.
b. What is the approximate probability that the sample has mean fuel cost between \$3.65 and \$3.67?
c. What is the approximate probability that the sample has a mean fuel cost that exceeds \$3.67?
d. How would the sampling distribution of $\bar{x}$ change if the sample size n were doubled from 100 to 200? How do your answers to parts **b.** and **c.** change when the sample size is doubled?