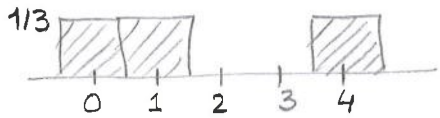
6.14

Consider the following probability distribution:

x	0	1	4
$p(x)$	1/3	1/3	1/3

Histogram of the mass fn $p(x)$:



a. Find μ and σ^2.
b. Find the sampling distribution of the sample mean $\bar{x}$ for a random sample of $n = 2$ measurements from this distribution.
~~**c.** Show that $\bar{x}$ is an unbiased estimator of μ. [Hint: Show that $E(\bar{x}) = \sum \bar{x}p(\bar{x}) = \mu$.]~~
d. Find the sampling distribution of the sample variance s^2 for a random sample of $n = 2$ measurements from this distribution.
~~**e.** Show that s^2 is an unbiased estimator for σ^2.~~

a. $\boxed{\mu = E(X)} = \sum_x x \cdot p(x) = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 4 \cdot \frac{1}{3} = \frac{1+4}{3} = \frac{5}{3} = \boxed{1.6\hat{6}}$

$\boxed{\sigma^2 = V(X)} = E(X^2) - \mu^2 = \frac{17}{3} - \frac{5^2}{3^2} = \frac{3 \cdot 17 - 25}{3^2} = \boxed{\frac{26}{9}}$

$E(X^2) = \sum_x x^2 \cdot p(x) = 0^2 \cdot \frac{1}{3} + 1^2 \cdot \frac{1}{3} + 4^2 \cdot \frac{1}{3} = \frac{1+16}{3} = \frac{17}{3}$

b. Possible samples of size $n=2$:

		Samples	Probability	$\bar{x}$	$s^2 = \frac{1}{n-1}\sum_i (x_i - \bar{x})^2$; $(n=2)$
0	0	(0, 0)	$(1/3)^2 = 1/9$	0	0
	1	(0, 1)	1/9	0.5	$0.5 = (0-0.5)^2 + (1-0.5)^2$
	4	(0, 4)	1/9	2	$8 = (0-2)^2 + (4-2)^2$
1	0	(1, 0)	1/9	0.5	0.5
	1	(1, 1)	1/9	1	0
	4	(1, 4)	1/9	2.5	$4.5 = (1-2.5)^2 + (4-2.5)^2$
4	0	(4, 0)	1/9	2	8
	1	(4, 1)	1/9	2.5	4.5
	4	(4, 4)	1/9	4	0

Sampling distribution of $\bar{x}$:

Values of $\bar{x}$	0	0.5	1	2	2.5	4
Probability	1/9	$\frac{1}{9} + \frac{1}{9} = \frac{2}{9}$	$\frac{1}{9}$	$\frac{1}{9} + \frac{1}{9} = \frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

d. Sampling distribution of s^2:

Values of s^2	0	0.5	4.5	8
Probability	$\frac{1}{9} \cdot 3 = \frac{3}{9} = \frac{1}{3}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$