6.38

$\mu = 3.65$

Cost of unleaded fuel. According to the American Automobile Association (AAA), the average cost of a gallon of regular unleaded fuel at gas stations in May 2014 was \$3.65 (*AAA Fuel Gauge Report*). Assume that the standard deviation of such costs is \$.15. Suppose that a random sample of $n = 100$ gas stations is selected from the population and the cost per gallon of regular unleaded fuel is determined for each. Consider $\bar{x}$, the sample mean cost per gallon.

$\sigma = 0.15$

a. Calculate $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$.
b. What is the approximate probability that the sample has mean fuel cost between \$3.65 and \$3.67?
c. What is the approximate probability that the sample has a mean fuel cost that exceeds \$3.67?
d. How would the sampling distribution of $\bar{x}$ change if the sample size n were doubled from 100 to 200? How do your answers to parts **b** and **c** change when the sample size is doubled?

a. $\mu_{\bar{X}} = E(\bar{X}) = 3.65 \qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.15}{\sqrt{100}} = 0.015$

b. $P\{3.65 < \bar{X} < 3.67\} \underset{CLT}{\simeq} P\left\{\frac{3.65-3.65}{0.015} < Z < \frac{3.67-3.65}{0.015}\right\}$

$\bar{X} \sim N(3.65,\ 0.015)$

$= P\{0 < Z < 1.33\} = 0.4082$

c. $P\{\bar{X} > 3.67\} \underset{CLT}{\simeq} P\{Z > 1.33\} = 0.5 - 0.4082$
$= 0.0918$

d. $n = 200 \longrightarrow \bar{X} \sim N\left(3.65,\ \frac{0.15}{\sqrt{200}} \simeq 0.011\right)$

$P\{3.65 < \bar{X} < 3.67\} \simeq P\{0 < Z < 1.82\} = 0.4656$

$P\{\bar{X} > 3.67\} \simeq P\{Z > 1.82\} = 0.5 - 0.4656 =$
$= 0.0344$