Basic Statistics and Probability (2018-19)
Science & Engineering Program Boston University-Faculty of Science UAM

Chapter 5: Continuous Random Variables

The following problems are from McClave, J. and Sincich, T. (2017), *Statistics*, 13th. edition, Pearson. The number preceding the exercise is the corresponding one from this textbook. Some of the problems may also appear in previous editions of the book, possibly with different numbering.

5.3 Suppose X is a random variable best described by a uniform probability distribution with $c = 10$ and $d = 30$.

a. Find $f(x)$.
b. Find the mean and standard deviation of x.
c. Graph $f(x)$, and locate μ and the interval $\mu \pm 2\sigma$ on the graph. Note that the probability that x assumes a value within the interval $\mu \pm 2\sigma$ is equal to 1.

5.4 Refer to Exercise 5.3. Find the following probabilities:

a. $P(10 \leq x \leq 25)$
b. $P(20 < x < 30)$
c. $P(x \geq 25)$
d. $P(x \leq 10)$
e. $P(x \leq 25)$
f. $P(20.5 \leq x \leq 25.5)$

5.13 Maintaining pipe wall temperature. Maintaining a constant temperature in a pipe wall in some hot process applications is critical. A new technique that utilizes bolt-on trace elements to maintain the temperature was presented in the *Journal of Heat Transfer* (Nov. 2000). Without bolt-on trace elements, the pipe wall temperature of a switch condenser used to produce plastic has a uniform distribution ranging from 260 to 290F. When several bolt-on trace elements are attached to the piping, the wall temperature is uniform from 278 to 285F.

a. Ideally, the pipe wall temperature should range between 280 and 284F. What is the probability that the temperature will fall into this ideal range when no bolt-on trace elements are used? when bolt-on trace elements are attached to the pipe?
b. When the temperature is 268F or lower, the hot liquid plastic hardens (or "plates"), causing a buildup in the piping. What is the probability of the plastic plating when no bolt-on trace elements are used? when bolt-on trace elements are attached to the pipe?

5.26 Find the following probabilities for the standard normal random variable z:

a. $P(z > 1.46)$
b. $P(z < -1.56)$
c. $P(.67 \leq z \leq 2.41)$
d. $P(-1.96 \leq z \leq -.33)$
e. $P(z \geq 0)$
f. $P(-2.33 < z < 1.50)$
g. $P(z \geq -2.33)$
h. $P(z < 2.33)$

5.28 Find a value z_0 of the standard normal random variable z such that

a. $P(z \leq z_0) = .0401$
b. $P(-z_0 \leq z \leq z_0) = .95$
c. $P(-z_0 \leq z \leq z_0) = .90$
d. $P(-z_0 \leq z \leq z_0) = .8740$
e. $P(-z_0 \leq z \leq 0) = .2967$
f. $P(-2 < z < z_0) = .9710$
g. $P(z \geq z_0) = .5$
h. $P(z \geq z_0) = .0057$

5.53 Range of womens heights. In *Chance* (Winter 2007), Yale Law School professor Ian Ayres published the re- sults of a study he conducted with his son and daughter on whether college students could estimate a range for women's heights. The students were shown a graph of a normal distribution of heights and were asked, "The average height of women over 20 years old in the United States is 64 inches. Using your intuition, please give your best estimate of the range of heights that would include C% of women over 20 years old. Please make sure that the center of the range is the average height of 64 inches." The value of C was randomly selected as 50%, 75%, 90%, 95%, or 99% for each student surveyed.

a. Give your estimate of the range for $C = 50\%$ of women's heights.
b. Give your estimate of the range for $C = 75\%$ of women's heights.
c. Give your estimate of the range for $C = 90\%$ of women's heights.
d. Give your estimate of the range for $C = 95\%$ of women's heights.
e. Give your estimate of the range for $C = 99\%$ of women's heights.
f. The standard deviation of heights for women over 20 years old is known to be 2.6 inches. Use this information to revise your answers to parts **a.–e.**.
g. Which value of C has the most accurate estimated range? (Note: The researchers found that college students were most accurate for $C = 90\%$ and $C = 95\%$.)

5.62 Examine the following sample data.

5.9	4.0	6.0	4.6	5.3	1.6	7.4
8.6	7.3	8.4	5.9	6.7	9.7	3.5
3.1	4.3	8.2	6.5	1.1	5.0	3.2
2.1	4.5	6.3	3.3	8.4	9.4	6.4

a. Construct a stem-and-leaf plot to assess whether the data are from an approximately normal distribution.
b. Compute s for the sample data.
c. Find the values of Q_L and Q_U and the value of s from part **b.** to assess whether the data come from an approximately normal distribution.
d. Generate a normal probability plot for the data, and use it to assess whether the data are approximately normal.

Indication: To load the data in R, you just have to write

```
X = c(5.9, 4.0, 6.0, 4.6, 5.3, 1.6, 7.4,
8.6, 7.3, 8.4, 5.9, 6.7, 9.7, 3.5,
3.1, 4.3, 8.2, 6.5, 1.1, 5.0, 3.2,
2.1, 4.5, 6.3, 3.3, 8.4, 9.4, 6.4)
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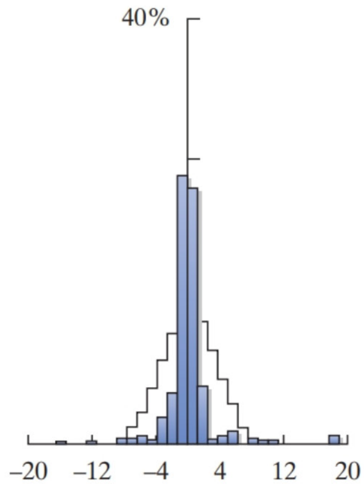
5.67 Estimating glacier elevations. Digital elevation models (DEMs) are now used to estimate elevations and slopes of remote regions. In *Arctic, Antarctic, and Alpine Research* (May 2004), geographers analyzed reading errors from maps produced by DEMs. Two readers of a DEM map of White Glacier (in Canada) estimated elevations at 400 points in the area. The difference between the elevation estimates of the two readers had a mean of $\mu = .28$ meter and a standard deviation of $\sigma = 1.6$ meters. A histogram of the difference (with a normal histogram superim- posed on the graph) is shown below.



“Uncertainty in digital elevation models of Axel Heiberg Island. Arctic Canada,”

a. On the basis of the histogram, the researchers con- cluded that the difference between elevation estimates is not normally distributed. Why?
b. Will the interval $\mu \pm 2\sigma$ contain more than 95%, exactly 95%, or less than 95% of the 400 elevation differences? Explain.

5.79 Suppose x is a binomial random variable with $p = .4$ and $n = 25$.

a. Would it be appropriate to approximate the probability distribution of x with a normal distribution? Explain.
b. Assuming that a normal distribution provides an adequate approximation to the distribution of x, what are the mean and variance of the approximating normal distribution?
c. Use Table I of Appendix B or statistical software to find the exact value of $P(x \geq 9)$.
d. Use the normal approximation to find $P(x \geq 9)$.

5.87 Analysis of bottled water. Refer to the report on whether bottled water is really purified water, presented in Exercise 4.72. Recall that the Natural Resources Defense Council found that 25% of bottled-water brands fill their bottles with just tap water. In a random sample of 70 bottled-water brands, let x be the number that contain tap water.

a. Find the mean of x.
b. Find the standard deviation of x.
c. Find the z-score for the value $x = 25$.
d. Find the approximate probability that 25 or more of the 70 sampled bottled-water brands will contain tap water.